# 11 номер – Д 2239

## Пример:

$\int_0^{\ln 2} xe^{-x}\, dx$

## Решение:

$I = \int_0^{\ln 2} xe^{-x} dx$

$I = \int u\, dv;\ \int u\, dv = uv - \int v\, du$
$u = x;\ du = dx$
$dv = e^{-x} dx;\ v = -e^{-x}$

$I = .\, x(-e^{-x})|_0^{\ln 2} - \int_0^{\ln 2} (-e^{-x}) dx = .\, -xe^{-x}|_0^{\ln 2} + \int_0^{\ln 2} e^{-x} dx$

$\int e^{-x} dx = -e^{-x}$
$I = .\, -xe^{-x}|_0^{\ln 2} + .\, -e^{-x}|_0^{\ln 2}$

$e^{-\ln 2} = \frac{1}{2}$
$I = -(\ln 2) \cdot \frac{1}{2} - \frac{1}{2} - (0 \cdot 1 - 1) = -\frac{\ln 2}{2} - \frac{1}{2} + 1 = \frac{1}{2} - \frac{\ln 2}{2}$

# 13 номер – Д 2241

## Пример:

$\int_0^{2\pi} x^2 \cos x\, dx$

## Решение:

$I = \int_0^{2\pi} x^2 \cos x\, dx$

$I = \int u\, dv;\ \int u\, dv = uv - \int v\, du$
$u = x^2;\ du = 2x\, dx$
$dv = \cos x\, dx;\ v = \sin x$

$I = .\, x^2 \sin x|_0^{2\pi} - \int_0^{2\pi} 2x \sin x\, dx = -2 \int_0^{2\pi} x \sin x\, dx$

$J = \int_0^{2\pi} x \sin x\, dx$
$J = \int u\, dv;\ \int u\, dv = uv - \int v\, du$
$u = x;\ du = dx$
$dv = \sin x\, dx;\ v = -\cos x$

$J = .\, -x \cos x|_0^{2\pi} + \int_0^{2\pi} \cos x\, dx = .\, -x \cos x|_0^{2\pi} + .\, \sin x|_0^{2\pi}$
$J = -2\pi \cdot 1 + 0 - 0 = -2\pi$

$I = -2J = -2(-2\pi) = 4\pi$

## 15 номер – Д 2244

### Пример:

$\int_0^{\sqrt{3}} x\,\mathrm{arccot} x\,dx$

### Решение:

$I = \int_0^{\sqrt{3}} x\,\mathrm{arccot}\,x\,dx$

$I = \int u\,dv;\ \int u\,dv = uv - \int v\,du$
$u = \mathrm{arccot}\,x;\ du = -\dfrac{1}{1+x^2}dx$
$dv = x\,dx;\ v = \dfrac{x^2}{2}$

$I =. \dfrac{x^2}{2}\mathrm{arccot}\,x|_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \dfrac{x^2}{2}(-\dfrac{1}{1+x^2})dx =. \dfrac{x^2}{2}\mathrm{arccot}\,x|_0^{\sqrt{3}} + \dfrac{1}{2}\int_0^{\sqrt{3}} \dfrac{x^2}{1+x^2}dx$

$\dfrac{x^2}{1+x^2} = 1 - \dfrac{1}{1+x^2}$
$I =. \dfrac{x^2}{2}\mathrm{arccot}\,x|_0^{\sqrt{3}} + \dfrac{1}{2}\int_0^{\sqrt{3}}(1 - \dfrac{1}{1+x^2})dx$
$\int \dfrac{dx}{1+x^2} = \arctan x$
$I =. \dfrac{x^2}{2}\mathrm{arccot}\,x|_0^{\sqrt{3}} + \dfrac{1}{2}.(x - \arctan x)|_0^{\sqrt{3}}$

$\mathrm{arccot}\sqrt{3} = \dfrac{\pi}{6};\ \arctan\sqrt{3} = \dfrac{\pi}{3}$
$I = \dfrac{3}{2}\cdot\dfrac{\pi}{6} + \dfrac{1}{2}(\sqrt{3} - \dfrac{\pi}{3}) = \dfrac{\pi}{4} + \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2} + \dfrac{\pi}{12}$

## 17 номер – Д 2246

### Пример:

$\int_0^a x^2\sqrt{a^2-x^2}\,dx$

### Решение:

$a > 0$
$I = \int_0^a x^2\sqrt{a^2-x^2}\,dx$

$x = a\sin t;\ dx = a\cos t\,dt;\ \sqrt{a^2-x^2} = a\cos t$
$x = 0 \implies t = 0;\ x = a \implies t = \dfrac{\pi}{2}$

$I = \int_0^{\pi/2}(a^2\sin^2 t)\cdot(a\cos t)\cdot(a\cos t)\,dt = a^4\int_0^{\pi/2}\sin^2 t\cos^2 t\,dt$

$\sin^2 t\cos^2 t = \frac{1}{4}\sin^2 2t;\ \sin^2 2t = \frac{1-\cos 4t}{2}$
$I = a^4\int_0^{\pi/2}\frac{1}{8}(1-\cos 4t)\,dt = \frac{a^4}{8}(t-\frac{\sin 4t}{4})\Big|_0^{\pi/2}$

$I = \frac{a^4}{8}(\frac{\pi}{2}-0) = \frac{\pi a^4}{16}$

# 19 номер – Д 2248

## Пример:

$\int_0^{\ln 2}\sqrt{e^x-1}\,dx$

## Решение:

$I = \int_0^{\ln 2}\sqrt{e^x-1}\,dx$

$t = e^x;\ dt = e^x dx;\ dx = \frac{dt}{t}$
$x = 0 \implies t = 1;\ x = \ln 2 \implies t = 2$

$I = \int_1^2\frac{\sqrt{t-1}}{t}\,dt$

$u = \sqrt{t-1};\ t = u^2+1;\ dt = 2u\,du$
$t = 1 \implies u = 0;\ t = 2 \implies u = 1$

$I = \int_0^1\frac{u}{u^2+1}\cdot 2u\,du = 2\int_0^1\frac{u^2}{u^2+1}\,du$
$\frac{u^2}{u^2+1} = 1-\frac{1}{u^2+1}$

$I = 2\int_0^1(1-\frac{1}{u^2+1})du = 2(u-\arctan u)\Big|_0^1$
$I = 2(1-\frac{\pi}{4}) = 2-\frac{\pi}{2}$

# 21 номер – Д 2269

## Пример:

$\int_{-1}^1\frac{xdx}{x^2+x+1}$

## Решение:

$I = \int_{-1}^1\frac{x}{x^2+x+1}dx$

$x = \frac{1}{2}(2x+1) - \frac{1}{2}$

$I = \frac{1}{2}\int_{-1}^{1} \frac{2x+1}{x^2+x+1}dx - \frac{1}{2}\int_{-1}^{1} \frac{dx}{x^2+x+1}$

$I_1 = \frac{1}{2}\int_{-1}^{1} \frac{2x+1}{x^2+x+1}dx = \frac{1}{2}.\ln(x^2+x+1)|_{-1}^{1} = \frac{1}{2}(\ln 3 - \ln 1) = \frac{1}{2}\ln 3$

$x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4}$

$I_2 = -\frac{1}{2}\int_{-1}^{1} \frac{dx}{(x+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

$\int \frac{dx}{(x-a)^2+b^2} = \frac{1}{b}\arctan\frac{x-a}{b}$

$I_2 = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}}.\arctan(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}})|_{-1}^{1} = -\frac{1}{\sqrt{3}}.\arctan(\frac{2x+1}{\sqrt{3}})|_{-1}^{1}$

$\arctan\frac{3}{\sqrt{3}} = \arctan\sqrt{3} = \frac{\pi}{3};\ \arctan\frac{-1}{\sqrt{3}} = -\frac{\pi}{6}$

$I_2 = -\frac{1}{\sqrt{3}}(\frac{\pi}{3} - (-\frac{\pi}{6})) = -\frac{1}{\sqrt{3}}\cdot\frac{\pi}{2} = -\frac{\pi}{2\sqrt{3}}$

$I = I_1 + I_2 = \frac{1}{2}\ln 3 - \frac{\pi}{2\sqrt{3}}$

## 23 номер – Д 2271

### Пример:

$\int_1^9 x\sqrt[3]{1-x}\,dx$

### Решение:

$I = \int_1^9 x\sqrt[3]{1-x}\,dx$

$t = 1-x;\ dt = -dx;\ x = 1-t$
$x = 1; t = 0;\ x = 9; t = -8$

$I = \int_0^{-8}(1-t)t^{\frac{1}{3}}(-dt) = \int_{-8}^{0}(1-t)t^{\frac{1}{3}}dt = \int_{-8}^{0}(t^{\frac{1}{3}} - t^{\frac{4}{3}})dt$

$I =.(\frac{3}{4}t^{\frac{4}{3}} - \frac{3}{7}t^{\frac{7}{3}})|_{-8}^{0} = -(\frac{3}{4}(-8)^{\frac{4}{3}} - \frac{3}{7}(-8)^{\frac{7}{3}})$

$\sqrt[3]{-8} = -2;\ (-8)^{\frac{4}{3}} = (\sqrt[3]{-8})^4 = (-2)^4 = 16;\ (-8)^{\frac{7}{3}} = (\sqrt[3]{-8})^7 = (-2)^7 = -128$

$I = -(\frac{3}{4}\cdot 16 - \frac{3}{7}\cdot(-128)) = -(12 + \frac{384}{7}) = -\frac{468}{7}$

## 25 номер – Д 2273

## Пример:

$\int_0^1 x^{15}\sqrt{1+3x^8}\,dx$

## Решение:

$I=\int_0^1 x^{15}\sqrt{1+3x^8}dx$

$u=1+3x^8;\ du=24x^7dx;\ x^7dx=\frac{1}{24}du$
$x^{15}dx=x^8\cdot x^7dx$

$x=0;\ u=1$
$x=1;\ u=4$

$x^8=\frac{u-1}{3}$
$I=\int_1^4 \frac{u-1}{3}\cdot\sqrt{u}\cdot\frac{1}{24}du=\frac{1}{72}\int_1^4(u-1)u^{\frac{1}{2}}du=\frac{1}{72}\int_1^4(u^{\frac{3}{2}}-u^{\frac{1}{2}})du$

$I=\frac{1}{72}(\frac{2}{5}u^{\frac{5}{2}}-\frac{2}{3}u^{\frac{3}{2}})\Big|_1^4=\frac{1}{72}(\frac{2}{5}(4^{\frac{5}{2}}-1)-\frac{2}{3}(4^{\frac{3}{2}}-1))$
$4^{\frac{5}{2}}=32;\ 4^{\frac{3}{2}}=8$
$I=\frac{1}{72}(\frac{2}{5}\cdot 31-\frac{2}{3}\cdot 7)=\frac{1}{72}(\frac{62}{5}-\frac{14}{3})=\frac{1}{72}\cdot\frac{116}{15}=\frac{29}{270}$

# 27 номер – Д 2275

## Пример:

$\int_0^{2\pi}\frac{dx}{(2+\cos x)(3+\cos x)}$

## Решение:

$I=\int_0^{2\pi}\frac{dx}{(2+\cos x)(3+\cos x)}$

$\frac{1}{(2+\cos x)(3+\cos x)}=\frac{1}{2+\cos x}-\frac{1}{3+\cos x}$
$I=I_1-I_2$
$I_1=\int_0^{2\pi}\frac{dx}{2+\cos x}$
$I_2=\int_0^{2\pi}\frac{dx}{3+\cos x}$

$\cos(\pi+t)=-\cos t$
$\int_0^{2\pi}\frac{dx}{a+\cos x}=2\int_0^{\pi}\frac{dx}{a+\cos x}\ (a>1)$

$I_1=2\int_0^{\pi}\frac{dx}{2+\cos x}$
$t=\tan\frac{x}{2}$

$\cos x = \dfrac{1-t^2}{1+t^2}$
$dx = \dfrac{2dt}{1+t^2}$
$x: 0 \to \pi;\ t: 0 \to +\infty$

$I_1 = 2\int_0^{+\infty} \dfrac{\frac{2dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}} = 2\int_0^{+\infty} \dfrac{2dt}{3+t^2} = 4\int_0^{+\infty} \dfrac{dt}{t^2+3}$
$\int \dfrac{dt}{t^2+a^2} = \dfrac{1}{a}\arctan\dfrac{t}{a}$
$I_1 = 4 \cdot \dfrac{1}{\sqrt{3}}\arctan\dfrac{t}{\sqrt{3}}\Big|_0^{+\infty} = 4 \cdot \dfrac{1}{\sqrt{3}}(\dfrac{\pi}{2} - 0) = \dfrac{2\pi}{\sqrt{3}}$

$I_2 = 2\int_0^{\pi} \dfrac{dx}{3+\cos x}$
$t = \tan\dfrac{x}{2}$
$\cos x = \dfrac{1-t^2}{1+t^2}$
$dx = \dfrac{2dt}{1+t^2}$
$x: 0 \to \pi;\ t: 0 \to +\infty$

$I_2 = 2\int_0^{+\infty} \dfrac{\frac{2dt}{1+t^2}}{3+\frac{1-t^2}{1+t^2}} = 2\int_0^{+\infty} \dfrac{2dt}{4+2t^2} = 2\int_0^{+\infty} \dfrac{dt}{t^2+2}$
$I_2 = 2 \cdot \dfrac{1}{\sqrt{2}}\arctan\dfrac{t}{\sqrt{2}}\Big|_0^{+\infty} = 2 \cdot \dfrac{1}{\sqrt{2}}(\dfrac{\pi}{2} - 0) = \dfrac{\pi}{\sqrt{2}}$

$I = \dfrac{2\pi}{\sqrt{3}} - \dfrac{\pi}{\sqrt{2}}$

# 29 номер – Д 2278

## Пример:

$\int_0^{\pi} (x \sin x)^2\, dx$

## Решение:

$I = \int_0^{\pi} (x\sin x)^2 dx = \int_0^{\pi} x^2 \sin^2 x\, dx$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$
$I = \dfrac{1}{2}\int_0^{\pi} x^2 dx - \dfrac{1}{2}\int_0^{\pi} x^2 \cos 2x\, dx$
$I = \dfrac{1}{2} \cdot \dfrac{x^3}{3}\Big|_0^{\pi} - \dfrac{1}{2}J = \dfrac{\pi^3}{6} - \dfrac{1}{2}J$

$J = \int_0^{\pi} x^2 \cos 2x\, dx$
$J = \int u\, dv;\ \ \int u\, dv = uv - \int v\, du$

$u = x^2;\ du = 2x\,dx$
$dv = \cos 2x\,dx;\ v = \frac{1}{2}\sin 2x$

$J =. \frac{x^2}{2}\sin 2x|_0^\pi - \int_0^\pi x\sin 2x\,dx = -K$

$K = \int_0^\pi x\sin 2x\,dx$
$K = \int u\,dv;\ \int u\,dv = uv - \int v\,du$
$u = x;\ du = dx$
$dv = \sin 2x\,dx;\ v = -\frac{1}{2}\cos 2x$

$K =. -\frac{x}{2}\cos 2x|_0^\pi + \frac{1}{2}\int_0^\pi \cos 2x\,dx$
$\int \cos 2x\,dx = \frac{1}{2}\sin 2x$
$K = -\frac{\pi}{2}\cdot 1 + \frac{1}{2}.\frac{1}{2}\sin 2x|_0^\pi = -\frac{\pi}{2}$

$J = -K = \frac{\pi}{2}$
$I = \frac{\pi^3}{6} - \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi^3}{6} - \frac{\pi}{4}$

# 40 номер – Д 2395

## Пример:

$v.p. \int_{-\infty}^{+\infty} \operatorname{arccot} x\,dx$

## Решение:

$I = \text{v.p.} \int_{-\infty}^{+\infty} \operatorname{arccot} x\,dx = \lim_{A\to+\infty} \int_{-A}^{A} \operatorname{arccot} x\,dx$

$\operatorname{arccot} x = \frac{\pi}{2} - \arctan x$
$I(A) = \int_{-A}^{A}(\frac{\pi}{2} - \arctan x)dx = \frac{\pi}{2}\int_{-A}^{A} dx - \int_{-A}^{A}\arctan x\,dx$

$\arctan x$ нечётная
$\int_{-A}^{A}\arctan x\,dx = 0$
$I(A) = \frac{\pi}{2}\cdot 2A = \pi A$

$I = \lim_{A\to+\infty} \pi A = +\infty$
v.p. интеграл расходится увы

# 42 номер – Д 2398

## Пример:

Площадь
$y = x^2; x + y = 2$

## Решение:

$y = x^2;\ y = 2 - x$

$x^2 = 2 - x$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x_1 = -2;\ x_2 = 1$

$S = \int_{-2}^{1} \left((2-x) - x^2\right) dx = \int_{-2}^{1} (2 - x - x^2)\, dx$
$S = (2x - \frac{x^2}{2} - \frac{x^3}{3})\Big|_{-2}^{1}$
$S = (2 - \frac{1}{2} - \frac{1}{3}) - (-4 - \frac{4}{2} + \frac{8}{3}) = \frac{7}{6} + \frac{10}{3} = \frac{9}{2}$

# 44 номер – Д 2400

## Пример:

Площадь
$y = |lgx|; y = 0; x = 0,1; x = 10;$

## Решение:

$S = \int_{0,1}^{10} |lgx|\, dx$
$lgx < 0\ (0 < x < 1);\ lgx > 0\ (x > 1)$
$S = \int_{0,1}^{1} (-lgx)\, dx + \int_{1}^{10} lgx\, dx$

$lgx = \frac{\ln x}{\ln 10}$
$\int lgx\, dx = \frac{1}{\ln 10} \int \ln x\, dx = \frac{1}{\ln 10}(x \ln x - x)$

$S_2 = \int_{1}^{10} lgx\, dx = \frac{1}{\ln 10}(x \ln x - x)\Big|_{1}^{10} = 10 - \frac{9}{\ln 10}$
$S_1 = \int_{0,1}^{1} (-lgx)\, dx = -\frac{1}{\ln 10}(x \ln x - x)\Big|_{0,1}^{1} = -(-\frac{1}{\ln 10} + 0,1 + \frac{0,1}{\ln 10}) = -0,1 + \frac{0,9}{\ln 10}$

$S = S_1 + S_2 = 9,9 - \frac{8,1}{\ln 10} = \frac{99}{10} - \frac{81}{10 \ln 10}$

# 46 номер – Д 2414

## Пример:

Площадь
$x = 2t - t^2; y = 2t^2 - t^3;$

## Решение:

$x = t(2-t);\ y = t^2(2-t)$
$t = 0;\ x = 0;\ y = 0$

$t = 2;\ x = 0;\ y = 0$

$$S = \frac{1}{2}\int_0^2 (x\frac{dy}{dt} - y\frac{dx}{dt})dt$$

$y = tx$
$\frac{dy}{dt} = x + t\frac{dx}{dt}$
$x\frac{dy}{dt} - y\frac{dx}{dt} = x(x + t\frac{dx}{dt}) - tx\frac{dx}{dt} = x^2$

$$S = \frac{1}{2}\int_0^2 x^2 dt = \frac{1}{2}\int_0^2 (t(2-t))^2 dt = \frac{1}{2}\int_0^2 (4t^2 - 4t^3 + t^4)dt$$
$$S = \frac{1}{2}(\frac{4}{3}t^3 - t^4 + \frac{1}{5}t^5)\Big|_0^2 = \frac{1}{2}(\frac{32}{3} - 16 + \frac{32}{5}) = \frac{1}{2}\cdot\frac{16}{15} = \frac{8}{15}$$

# 48 номер – Д 2418

## Пример:

Площадь
$r^2 = a^2\cos 2\phi\,(\textit{лемниската})$

## Решение:

$r^2 = a^2\cos 2\phi$
$\cos 2\phi \geq 0;\ \phi \in [-\frac{\pi}{4}; \frac{\pi}{4}]$ (одна петля)

$$S_1 = \frac{1}{2}\int_{-\pi/4}^{\pi/4} r^2 d\phi = \frac{1}{2}\int_{-\pi/4}^{\pi/4} a^2\cos 2\phi\, d\phi$$
$$S_1 = \frac{a^2}{2}\cdot\frac{1}{2}\sin 2\phi\Big|_{-\pi/4}^{\pi/4} = \frac{a^2}{4}(\sin\frac{\pi}{2} - \sin(-\frac{\pi}{2})) = \frac{a^2}{4}(1 - (-1)) = \frac{a^2}{2}$$

$S = 2S_1 = a^2$

# 50 номер – Д 2431

## Пример:

Длины дуг кривой
$y = x^{\frac{3}{2}}; (0 \leq x \leq 4)$

## Решение:

$$l = \int_0^4 \sqrt{1 + (y')^2}\,dx$$
$y = x^{\frac{3}{2}};\ y' = \frac{3}{2}x^{\frac{1}{2}}$

$(y')^2 = \frac{9}{4}x$

$l = \int\limits_0^4 \sqrt{1 + \frac{9}{4}x}\, dx = \frac{1}{2}\int\limits_0^4 \sqrt{9x+4}\, dx$

$u = 9x + 4;\ du = 9dx;\ dx = \frac{du}{9}$

$x = 0;\ u = 4$

$x = 4;\ u = 40$

$l = \frac{1}{2}\int\limits_4^{40} \sqrt{u} \cdot \frac{du}{9} = \frac{1}{18}\int\limits_4^{40} u^{\frac{1}{2}} du = \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}}\Big|_4^{40} = \frac{1}{27}(40^{\frac{3}{2}} - 4^{\frac{3}{2}})$

$40^{\frac{3}{2}} = 40\sqrt{40} = 80\sqrt{10};\ 4^{\frac{3}{2}} = 8$

$l = \frac{1}{27}(80\sqrt{10} - 8) = \frac{8}{27}(10\sqrt{10} - 1)$

## 52 номер – Д 2433

### Пример:

Длины дуг кривой

$y = a\cosh\frac{x}{a}$; от точки A(0,a) до точки B(b,h)

### Решение:

$a > 0$

$l = \int\limits_0^b \sqrt{1 + (y')^2}\, dx$

$y = a\cosh\frac{x}{a}$

$y' = a(\cosh\frac{x}{a})' = a \cdot \sinh\frac{x}{a} \cdot \frac{1}{a} = \sinh\frac{x}{a}$

$1 + (y')^2 = 1 + \sinh^2\frac{x}{a} = \cosh^2\frac{x}{a}$

$\sqrt{1 + (y')^2} = \cosh\frac{x}{a}$

$l = \int\limits_0^b \cosh\frac{x}{a}\, dx = a\sinh\frac{x}{a}\Big|_0^b = a\sinh\frac{b}{a}$

$h = y(b) = a\cosh\frac{b}{a}$

$\sinh\frac{b}{a} = \sqrt{\cosh^2\frac{b}{a} - 1} = \sqrt{(\frac{h}{a})^2 - 1} = \frac{\sqrt{h^2 - a^2}}{a}$

$l = a\sinh\frac{b}{a} = \sqrt{h^2 - a^2}$

## 54 номер – Д 2462

### Пример:

Объём
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1; z=\frac{c}{a}x; z=0;$$

## Решение:

$$V=\iiint\limits_{(V)} dV=\iint\limits_{D}(z_{\text{верх}}-z_{\text{низ}})\,dS$$

$$D:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$$
$$z_{\text{низ}}=0;\ z_{\text{верх}}=\frac{c}{a}x$$

$$z_{\text{верх}}\ge z_{\text{низ}};\ \frac{c}{a}x\ge 0;\ x\ge 0$$
$$D_1=D\cap\{x\ge 0\}$$

$$V=\iint\limits_{D_1}\frac{c}{a}x\,dS=\frac{c}{a}\iint\limits_{D_1}x\,dS$$

$$x=ar\cos t;\ y=br\sin t;\ 0\le r\le 1;\ -\frac{\pi}{2}\le t\le\frac{\pi}{2}$$
$$dS=ab\,r\,dr\,dt$$

$$\iint\limits_{D_1}x\,dS=\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int\limits_{0}^{1}(ar\cos t)\,ab\,r\,dr\,dt=a^2b\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\,dt\int\limits_{0}^{1}r^2\,dr$$
$$\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\,dt=.\sin t|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=2;\ \int\limits_{0}^{1}r^2\,dr=.\frac{r^3}{3}|_0^1=\frac{1}{3}$$
$$\iint\limits_{D_1}x\,dS=a^2b\cdot 2\cdot\frac{1}{3}=\frac{2a^2b}{3}$$

$$V=\frac{c}{a}\cdot\frac{2a^2b}{3}=\frac{2abc}{3}$$

# 56 номер – Д 2464

## Пример:

Объём
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1; z=\pm c$$

## Решение:

$$V=\int\limits_{-c}^{c}S(z)\,dz$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{z^2}{c^2}$$

$$S(z) = \pi \cdot a\sqrt{1 + \frac{z^2}{c^2}} \cdot b\sqrt{1 + \frac{z^2}{c^2}} = \pi ab(1 + \frac{z^2}{c^2})$$

$$V = \int\limits_{-c}^{c} \pi ab(1 + \frac{z^2}{c^2})dz = \pi ab(\int\limits_{-c}^{c} dz + \frac{1}{c^2} \int\limits_{-c}^{c} z^2 dz)$$
$$\int\limits_{-c}^{c} dz = 2c; \quad \int\limits_{-c}^{c} z^2 dz =. \frac{z^3}{3} |_{-c}^{c} = \frac{2c^3}{3}$$
$$V = \pi ab(2c + \frac{1}{c^2} \cdot \frac{2c^3}{3}) = \pi ab(2c + \frac{2c}{3}) = \frac{8\pi abc}{3}$$

# 58 номер – Д 2666

АЦЦЦКKИИИИИЙ НОМЕР он ещё и последний и именно поэтому я его делать НЕ БУДУ :D