60 номер – Д 1853

Пример:

$$\int \frac{xdx}{\sqrt{5+x-x^2}}$$

Решение:

$$f(x) = 5 + x - x^2$$
$$f'(x) = 1 - 2x$$

$$-\frac{1}{2}f'(x) = x - \frac{1}{2}$$
$$x = -\frac{1}{2}f'(x) + \frac{1}{2}$$

$$I = \int \frac{x}{\sqrt{f(x)}}dx = -\frac{1}{2}\int \frac{f'(x)}{\sqrt{f(x)}}dx + \frac{1}{2}\int \frac{dx}{\sqrt{f(x)}}$$

$$I = I_1 + I_2$$

$$u = f(x);\ du = f'(x)dx$$
$$I_1 = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\sqrt{u} = -\sqrt{f(x)}$$

$$f(x) = 5 + x - x^2 = \frac{21}{4} - (x - \frac{1}{2})^2$$

$$t = x - \frac{1}{2};\ dt = dx$$
$$I_2 = \frac{1}{2}\int \frac{dt}{\sqrt{\frac{21}{4} - t^2}} = \frac{1}{2}\arcsin(\frac{t}{\sqrt{21}/2}) = \frac{1}{2}\arcsin(\frac{2x-1}{\sqrt{21}})$$

$$I = -\sqrt{5+x-x^2} + \frac{1}{2}\arcsin(\frac{2x-1}{\sqrt{21}}) + C$$

61 номер – Д 1903

Пример:

$$\int \frac{x^3}{(x-1)^{100}}dx$$

Решение:

$$I = \int \frac{x^3}{(x-1)^{100}}dx$$

$t = x - 1;\ x = t + 1;\ dt = dx$

$I = \int \frac{(t+1)^3}{t^{100}} dt = \int \frac{t^3 + 3t^2 + 3t + 1}{t^{100}} dt = \int (t^{-97} + 3t^{-98} + 3t^{-99} + t^{-100}) dt$

$$I = -\frac{1}{96} t^{-96} - \frac{3}{97} t^{-97} - \frac{3}{98} t^{-98} - \frac{1}{99} t^{-99} + C$$

$$I = -\frac{1}{96(x-1)^{96}} - \frac{3}{97(x-1)^{97}} - \frac{3}{98(x-1)^{98}} - \frac{1}{99(x-1)^{99}} + C$$

62 номер – Д 1905

Пример:

$$\int \frac{x^3 dx}{x^8 + 3}$$

Решение:

$$I = \int \frac{x^3}{x^8 + 3} dx$$

$t = x^4;\ dt = 4x^3 dx;\ x^3 dx = \frac{1}{4} dt$

$I = \frac{1}{4} \int \frac{dt}{t^2 + 3} = \frac{1}{4} \cdot \frac{1}{\sqrt{3}} \arctan(\frac{t}{\sqrt{3}}) + C$

$$I = \frac{1}{4\sqrt{3}} \arctan(\frac{x^4}{\sqrt{3}}) + C$$

63 номер – Д 1907

Пример:

$$\int \frac{x^4 - 3}{x(x^8 + 3x^4 + 2)}$$

Решение:

$I = \int \frac{x^4 - 3}{x(x^8 + 3x^4 + 2)} dx$

$x^8 + 3x^4 + 2 = (x^4)^2 + 3x^4 + 2 = (x^4 + 1)(x^4 + 2)$

$t = x^4;\ dt = 4x^3 dx;\ \frac{dt}{t} = 4\frac{dx}{x};\ \frac{dx}{x} = \frac{1}{4}\frac{dt}{t}$

$I = \int \frac{x^4 - 3}{x(x^4 + 1)(x^4 + 2)} dx = \int \frac{t - 3}{(t+1)(t+2)} \frac{dx}{x} = \frac{1}{4} \int \frac{t - 3}{t(t+1)(t+2)} dt$

$\frac{t - 3}{t(t+1)(t+2)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{t+2}$

$t - 3 = A(t+1)(t+2) + Bt(t+2) + Ct(t+1)$

$A=-\frac{3}{2};\ B=4;\ C=-\frac{5}{2}$

$I=\frac{1}{4}\int(-\frac{3}{2}\frac{1}{t}+4\frac{1}{t+1}-\frac{5}{2}\frac{1}{t+2})dt$
$I=-\frac{3}{8}\ln|t|+\ln|t+1|-\frac{5}{8}\ln|t+2|+C$
$\ln|t|=\ln(x^4)=4\ln|x|$

$I=\ln(x^4+1)-\frac{3}{2}\ln|x|-\frac{5}{8}\ln(x^4+2)+C$

64 номер – Д 1909

Пример:

$\int\frac{x^{11}dx}{x^8+3x^4+2}$

Решение:

$I=\int\frac{x^{11}}{x^8+3x^4+2}dx$

$t=x^4;\ dt=4x^3dx;\ x^{11}dx=x^8\cdot x^3dx=t^2\cdot\frac{1}{4}dt$
$x^8+3x^4+2=t^2+3t+2=(t+1)(t+2)$

$I=\frac{1}{4}\int\frac{t^2}{t^2+3t+2}dt=\frac{1}{4}\int(1-\frac{3t+2}{(t+1)(t+2)})dt$

$\frac{3t+2}{(t+1)(t+2)}=\frac{A}{t+1}+\frac{B}{t+2}$
$3t+2=A(t+2)+B(t+1)=(A+B)t+(2A+B)$
$A=-1;\ B=4$

$I=\frac{1}{4}\int(1+\frac{1}{t+1}-\frac{4}{t+2})dt=\frac{1}{4}(t+\ln(t+1)-4\ln(t+2))+C$

$I=\frac{x^4}{4}+\frac{1}{4}\ln(x^4+1)-\ln(x^4+2)+C$

65 номер – Д 1910

Пример:

$\int\frac{x^9dx}{(x^{10}+2x^5+2)^2}$

Решение:

$$I = \int \frac{x^9}{(x^{10} + 2x^5 + 2)^2} dx$$

$$t = x^5;\ dt = 5x^4 dx;\ x^9 dx = x^5 \cdot x^4 dx = t \cdot \frac{1}{5} dt$$
$$x^{10} + 2x^5 + 2 = t^2 + 2t + 2$$

$$I = \frac{1}{5} \int \frac{t}{(t^2 + 2t + 2)^2} dt = \frac{1}{5} \int \left(\frac{t+1}{(t^2 + 2t + 2)^2} - \frac{1}{(t^2 + 2t + 2)^2}\right) dt$$
$$I = \frac{1}{5}(I_1 - I_2)$$

$$u = t^2 + 2t + 2;\ du = (2t + 2)dt = 2(t + 1)dt$$
$$I_1 = \int \frac{t+1}{(t^2 + 2t + 2)^2} dt = \frac{1}{2} \int \frac{du}{u^2} = -\frac{1}{2u} = -\frac{1}{2(t^2 + 2t + 2)}$$

$$t^2 + 2t + 2 = (t + 1)^2 + 1$$
$$s = t + 1;\ ds = dt$$
$$I_2 = \int \frac{dt}{(t^2 + 2t + 2)^2} = \int \frac{ds}{(s^2 + 1)^2}$$

$$\left(\frac{s}{s^2 + 1}\right)' = \frac{1 - s^2}{(s^2 + 1)^2}$$
$$\frac{1}{(s^2 + 1)^2} = \frac{1 - s^2}{(s^2 + 1)^2} + \frac{s^2}{(s^2 + 1)^2} = \left(\frac{s}{s^2 + 1}\right)' + \left(\frac{1}{s^2 + 1} - \frac{1}{(s^2 + 1)^2}\right)$$
$$2 \int \frac{ds}{(s^2 + 1)^2} = \frac{s}{s^2 + 1} + \arctan s$$
$$I_2 = \frac{1}{2}\left(\frac{s}{s^2 + 1} + \arctan s\right) = \frac{1}{2}\left(\frac{t+1}{t^2 + 2t + 2} + \arctan(t + 1)\right)$$

$$I = \frac{1}{5}\left(-\frac{1}{2(t^2 + 2t + 2)} - \frac{1}{2}\left(\frac{t+1}{t^2 + 2t + 2} + \arctan(t + 1)\right)\right) + C$$
$$I = -\frac{t+2}{10(t^2 + 2t + 2)} - \frac{1}{10} \arctan(t + 1) + C$$

$$I = -\frac{x^5 + 2}{10(x^{10} + 2x^5 + 2)} - \frac{1}{10} \arctan(x^5 + 1) + C$$

66 номер – Д 1913

Пример:

$$\int \frac{dx}{x(x^{10} + 2)}$$

Решение:

$$I = \int \frac{dx}{x(x^{10} + 2)}$$

$t = x^{10};\ dt = 10x^9dx;\ dx = \frac{dt}{10x^9}$

$I = \int \frac{\frac{dt}{10x^9}}{x(t+2)} = \frac{1}{10}\int \frac{dt}{x^{10}(t+2)} = \frac{1}{10}\int \frac{dt}{t(t+2)}$

$\frac{1}{t(t+2)} = \frac{A}{t} + \frac{B}{t+2}$

$1 = A(t+2) + Bt = (A+B)t + 2A$

$A = \frac{1}{2};\ B = -\frac{1}{2}$

$I = \frac{1}{10}\int(\frac{1}{2t} - \frac{1}{2(t+2)})dt = \frac{1}{20}(\ln|t| - \ln|t+2|) + C$

$I = \frac{1}{20}\ln|\frac{x^{10}}{x^{10}+2}| + C$

67 номер – Д 1915

Пример:

$\int \frac{1-x^7}{x(1+x^7)}\,dx$

Решение:

$I = \int \frac{1-x^7}{x(1+x^7)}dx$

$\frac{1-x^7}{x(1+x^7)} = \frac{1+x^7}{x(1+x^7)} - \frac{2x^7}{x(1+x^7)} = \frac{1}{x} - \frac{2x^6}{1+x^7}$

$I = \int \frac{dx}{x} - 2\int \frac{x^6}{1+x^7}dx$

$u = 1+x^7;\ du = 7x^6dx$

$\int \frac{x^6}{1+x^7}dx = \frac{1}{7}\int \frac{du}{u} = \frac{1}{7}\ln|u|$

$I = \ln|x| - \frac{2}{7}\ln|1+x^7| + C$

68 номер – Д 1916

Пример:

$\int \frac{x^4-1}{x(x^3-5)(x^5-5x+1)}\,dx$

Решение:

$$I = \int \frac{x^4 - 1}{x(x^3-5)(x^5-5x+1)}dx$$

$$\frac{x^4 - 1}{x(x^3-5)(x^5-5x+1)} = \frac{1}{5x} + \frac{187x^2 + 1575x + 1250}{7190(x^3-5)} - \frac{325x^4 + 315x^3 + 250x^2 + 187x - 1488}{1438(x^5-5x+1)}$$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{1}{5}\int \frac{dx}{x} = \frac{1}{5}\ln|x|$$

$$I_2 = \frac{1}{7190}\int \frac{187x^2 + 1575x + 1250}{x^3-5}dx = \frac{1}{7190}(\frac{187}{3}\int \frac{3x^2}{x^3-5}dx + \int \frac{1575x+1250}{x^3-5}dx)$$

$$u = x^3 - 5;\ du = 3x^2dx$$

$$\frac{187}{3}\int \frac{3x^2}{x^3-5}dx = \frac{187}{3}\ln|x^3-5|$$

$$a = \sqrt[3]{5};\ x^3 - 5 = (x-a)(x^2+ax+a^2)$$

$$\frac{1575x+1250}{x^3-5} = \frac{A}{x-a} + \frac{Bx+C}{x^2+ax+a^2}$$

$$A = \frac{1575a+1250}{3a^2};\ B = -A;\ C = \frac{1575a-2500}{3a}$$

$$\int \frac{A}{x-a}dx = A\ln|x-a|$$

$$\int \frac{Bx+C}{x^2+ax+a^2}dx = \frac{B}{2}\ln(x^2+ax+a^2) + (C - \frac{Ba}{2})\int \frac{dx}{x^2+ax+a^2}$$

$$x^2+ax+a^2 = (x+\frac{a}{2})^2 + \frac{3a^2}{4}$$

$$\int \frac{dx}{x^2+ax+a^2} = \frac{2}{a\sqrt{3}}\arctan(\frac{2x+a}{a\sqrt{3}})$$

$$I_2 = \frac{1}{7190}(\frac{187}{3}\ln|x^3-5| + A\ln|x-a| + \frac{B}{2}\ln(x^2+ax+a^2) + (C-\frac{Ba}{2})\frac{2}{a\sqrt{3}}\arctan(\frac{2x+a}{a\sqrt{3}}))$$

$$I_3 = -\frac{1}{1438}\int \frac{325x^4+315x^3+250x^2+187x-1488}{x^5-5x+1}dx$$

$$P(x) = x^5-5x+1;\ P'(x) = 5x^4-5$$

$r_1, \ldots, r_5$ — корни $P(x) = 0$

$$\frac{325x^4+315x^3+250x^2+187x-1488}{P(x)} = \sum_{k=1}^{5}\frac{325r_k^4+315r_k^3+250r_k^2+187r_k-1488}{P'(r_k)}\cdot\frac{1}{x-r_k}$$

$$I_3 = -\frac{1}{1438}\sum_{k=1}^{5}\frac{325r_k^4+315r_k^3+250r_k^2+187r_k-1488}{5r_k^4-5}\ln(x-r_k)$$

$$I = \frac{1}{5}\ln|x| + I_2 + I_3 + C$$

69 номер – Д 1917

Пример:

$\int \frac{x^2+1}{x^4+x^2+1}\,dx$

Решение:

$I = \int \frac{x^2+1}{x^4+x^2+1} dx$

$x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$
$x^2 + 1 = \frac{1}{2}[(x^2 - x + 1) + (x^2 + x + 1)]$

$I = \frac{1}{2}\int \frac{dx}{x^2 - x + 1} + \frac{1}{2}\int \frac{dx}{x^2 + x + 1} = I_1 + I_2$

$x^2 - x + 1 = (x - \frac{1}{2})^2 + \frac{3}{4}$
$t = x - \frac{1}{2};\ dt = dx$
$I_1 = \frac{1}{2}\int \frac{dt}{t^2 + (\frac{\sqrt{3}}{2})^2} = \frac{1}{\sqrt{3}}\arctan(\frac{2t}{\sqrt{3}}) = \frac{1}{\sqrt{3}}\arctan(\frac{2x-1}{\sqrt{3}})$

$x^2 + x + 1 = (x + \frac{1}{2})^2 + \frac{3}{4}$
$s = x + \frac{1}{2};\ ds = dx$
$I_2 = \frac{1}{2}\int \frac{ds}{s^2 + (\frac{\sqrt{3}}{2})^2} = \frac{1}{\sqrt{3}}\arctan(\frac{2s}{\sqrt{3}}) = \frac{1}{\sqrt{3}}\arctan(\frac{2x+1}{\sqrt{3}})$

$I = \frac{1}{\sqrt{3}}(\arctan(\frac{2x-1}{\sqrt{3}}) + \arctan(\frac{2x+1}{\sqrt{3}})) + C$

70 номер – Д 1921

да может ну это... ну не надо?

71 номер – Д 1971

Пример:

$\int \frac{dx}{\sqrt{x^2+1} - \sqrt{x^2-1}}$

Решение:

$x^2 - 1 \geq 0;\ |x| \geq 1;$

$I = \int \frac{dx}{\sqrt{x^2+1} - \sqrt{x^2-1}} \cdot \frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}} = \int \frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{(x^2+1) - (x^2-1)} dx$
$I = \frac{1}{2}\int (\sqrt{x^2+1} + \sqrt{x^2-1}) dx = \frac{1}{2}\int \sqrt{x^2+1}\,dx + \frac{1}{2}\int \sqrt{x^2-1}\,dx$
$I = I_1 + I_2$

$I_1 = \frac{1}{2}\int\sqrt{x^2+1}\,dx;\ \sqrt{x^2+1} = r$
$\int r\,dx = \frac{1}{2}(xr + \ln(x+r))$
$I_1 = \frac{1}{2}\cdot\frac{1}{2}(x\sqrt{x^2+1} + \ln|x+\sqrt{x^2+1}|) = \frac{1}{4}x\sqrt{x^2+1} + \frac{1}{4}\ln|x+\sqrt{x^2+1}|$

$I_2 = \frac{1}{2}\int\sqrt{x^2-1}\,dx;\ \sqrt{x^2-1} = s$
$\int s\,dx = \frac{1}{2}(xs - \ln|x+s|)$
$I_2 = \frac{1}{2}\cdot\frac{1}{2}(x\sqrt{x^2-1} - \ln|x+\sqrt{x^2-1}|) = \frac{1}{4}x\sqrt{x^2-1} - \frac{1}{4}\ln|x+\sqrt{x^2-1}|$

$I = \frac{x}{4}(\sqrt{x^2+1} + \sqrt{x^2-1}) + \frac{1}{4}\ln|\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2-1}}| + C$

72 номер – Д 1972

Пример:

$\int\frac{xdx}{(1-x^3)\sqrt{1-x^2}}$

Решение:

$I = \int\frac{x}{(1-x^3)\sqrt{1-x^2}}dx$

$\frac{1}{1-x^3} = \frac{1}{3(1-x)} + \frac{x+2}{3(x^2+x+1)}$
$I = \frac{1}{3}\int\frac{x}{(1-x)\sqrt{1-x^2}}dx + \frac{1}{3}\int\frac{x(x+2)}{(x^2+x+1)\sqrt{1-x^2}}dx$
$I = I_1 + I_2$

$x = \cos t;\ dx = -\sin t\,dt;\ \sqrt{1-x^2} = \sin t$
$I_1 = \frac{1}{3}\int\frac{\cos t}{(1-\cos t)\sin t}(-\sin t\,dt) = -\frac{1}{3}\int\frac{\cos t}{1-\cos t}dt$
$\frac{\cos t}{1-\cos t} = -1 + \frac{1}{1-\cos t}$
$I_1 = -\frac{1}{3}\int(-1 + \frac{1}{1-\cos t})dt = \frac{t}{3} - \frac{1}{3}\int\frac{dt}{1-\cos t}$
$1 - \cos t = 2\sin^2\frac{t}{2}$
$\int\frac{dt}{1-\cos t} = \int\frac{dt}{2\sin^2(t/2)} = -\cot\frac{t}{2}$
$I_1 = \frac{t}{3} + \frac{1}{3}\cot\frac{t}{2}$

$I_2 = \frac{1}{3}\int\frac{\cos t(\cos t+2)}{(\cos^2 t+\cos t+1)\sin t}(-\sin t\,dt) = -\frac{1}{3}\int\frac{\cos t(\cos t+2)}{\cos^2 t+\cos t+1}dt$
$\cos t(\cos t+2) = \cos^2 t + 2\cos t = (\cos^2 t+\cos t+1) + (\cos t - 1)$

$$I_2 = -\frac{1}{3}\int(1+\frac{\cos t - 1}{\cos^2 t + \cos t + 1})dt = -\frac{t}{3} - \frac{1}{3}\int\frac{\cos t - 1}{\cos^2 t + \cos t + 1}dt$$

$$I = I_1 + I_2 = \frac{1}{3}\cot\frac{t}{2} - \frac{1}{3}\int\frac{\cos t - 1}{\cos^2 t + \cos t + 1}dt$$

$$u = \tan\frac{t}{2};\ dt = \frac{2\,du}{1+u^2};\ \ \cos t = \frac{1-u^2}{1+u^2}$$
$$\cos t - 1 = -\frac{2u^2}{1+u^2}$$
$$\cos^2 t + \cos t + 1 = \frac{u^4+3}{(1+u^2)^2}$$
$$\frac{\cos t - 1}{\cos^2 t + \cos t + 1}dt = -\frac{4u^2}{u^4+3}du$$

$$I = \frac{1}{3}\cot\frac{t}{2} + \frac{4}{3}\int\frac{u^2}{u^4+3}du$$

$$p = \sqrt[4]{3}$$
$$u^4 + 3 = u^4 + p^4 = (u^2 - \sqrt{2}\,pu + \sqrt{3})(u^2 + \sqrt{2}\,pu + \sqrt{3})$$
$$\frac{u^2}{u^4+3} = \frac{\sqrt{2}\,p^3}{12}(\frac{u}{u^2 - \sqrt{2}\,pu + \sqrt{3}} - \frac{u}{u^2 + \sqrt{2}\,pu + \sqrt{3}})$$

$$\int\frac{u}{u^2 + \sqrt{2}\,pu + \sqrt{3}}du = \frac{1}{2}\ln(u^2 + \sqrt{2}\,pu + \sqrt{3}) - \arctan(\frac{\sqrt{2}}{p}u + 1)$$
$$\int\frac{u}{u^2 - \sqrt{2}\,pu + \sqrt{3}}du = \frac{1}{2}\ln(u^2 - \sqrt{2}\,pu + \sqrt{3}) + \arctan(\frac{\sqrt{2}}{p}u - 1)$$

$$\int\frac{u^2}{u^4+3}du = \frac{\sqrt{2}\,p^3}{24}\ln(\frac{u^2 - \sqrt{2}\,pu + \sqrt{3}}{u^2 + \sqrt{2}\,pu + \sqrt{3}}) + \frac{\sqrt{2}\,p^3}{12}(\arctan(\frac{\sqrt{2}}{p}u - 1) + \arctan(\frac{\sqrt{2}}{p}u + 1)) + C$$

$$\sqrt{2}\,p^3 = \frac{3\sqrt{2}}{p}$$
$$I = \frac{1}{3}\cot\frac{t}{2} + \frac{\sqrt{2}}{6p}\ln(\frac{u^2 - \sqrt{2}\,pu + \sqrt{3}}{u^2 + \sqrt{2}\,pu + \sqrt{3}}) + \frac{\sqrt{2}}{3p}(\arctan(\frac{\sqrt{2}}{p}u - 1) + \arctan(\frac{\sqrt{2}}{p}u + 1)) + C$$

$$\cot\frac{t}{2} = \frac{1+\cos t}{\sin t} = \frac{1+x}{\sqrt{1-x^2}}$$
$$u = \tan\frac{t}{2} = \frac{\sin t}{1+\cos t} = \frac{\sqrt{1-x^2}}{1+x}$$

$$I = \frac{1+x}{3\sqrt{1-x^2}} + \frac{\sqrt{2}}{6\sqrt[4]{3}}\ln(\frac{u^2 - \sqrt{2}\sqrt[4]{3}\,u + \sqrt{3}}{u^2 + \sqrt{2}\sqrt[4]{3}\,u + \sqrt{3}}) + \frac{\sqrt{2}}{3\sqrt[4]{3}}(\arctan(\frac{\sqrt{2}}{\sqrt[4]{3}}u - 1) + \arctan(\frac{\sqrt{2}}{\sqrt[4]{3}}u + 1)) + C$$
$$u = \frac{\sqrt{1-x^2}}{1+x}$$

73 номер – Д 1973

Пример:

$$\int \frac{dx}{\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}}$$

Решение:

$$-1 \le x \le 1$$

$$I = \int \frac{dx}{\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}}$$

$$x = \cos 2t;\ dx = -2\sin 2t\,dt$$
$$\sqrt{1-x} = \sqrt{1-\cos 2t} = \sqrt{2}\sin t$$
$$\sqrt{1+x} = \sqrt{1+\cos 2t} = \sqrt{2}\cos t$$

$$I = \int \frac{-2\sin 2t\,dt}{\sqrt{2}(1+\sin t+\cos t)} = -\sqrt{2}\int \frac{\sin 2t}{1+\sin t+\cos t}dt$$

$$(\sin t+\cos t)^2 = 1+2\sin t\cos t = 1+\sin 2t$$
$$\sin 2t = (\sin t+\cos t)^2 - 1$$

$$\frac{\sin 2t}{1+\sin t+\cos t} = \frac{(\sin t+\cos t)^2-1}{1+\sin t+\cos t} = \sin t+\cos t-1$$

$$I = -\sqrt{2}\int(\sin t+\cos t-1)dt = -\sqrt{2}(-\cos t+\sin t-t)+C$$
$$I = \sqrt{2}(\cos t-\sin t+t)+C$$

$$x = \cos 2t \implies t = \frac{1}{2}\arccos x$$
$$\cos t = \sqrt{\frac{1+\cos 2t}{2}} = \sqrt{\frac{1+x}{2}};\ \sin t = \sqrt{\frac{1-\cos 2t}{2}} = \sqrt{\frac{1-x}{2}}$$

$$I = \sqrt{1+x}-\sqrt{1-x}+\frac{1}{\sqrt{2}}\arccos x + C$$

74 номер – Д 1974

Пример:

$$\int \frac{x+\sqrt{1+x+x^2}}{1+x+\sqrt{1+x+x^2}}\,dx$$

Решение:

$$I = \int \frac{x+\sqrt{1+x+x^2}}{1+x+\sqrt{1+x+x^2}}dx$$
$$S = \sqrt{1+x+x^2}$$

$$\frac{x+S}{1+x+S} = 1-\frac{1}{1+x+S}$$
$$I = \int dx - \int \frac{dx}{1+x+S} = x - J$$

$$J=\int\frac{dx}{1+x+S}\cdot\frac{1+x-S}{1+x-S}=\int\frac{1+x-S}{(1+x)^2-S^2}dx$$
$$(1+x)^2-S^2=(1+2x+x^2)-(1+x+x^2)=x$$
$$J=\int\frac{1+x-S}{x}dx=\int(\frac{1}{x}+1-\frac{S}{x})dx=\ln|x|+x-K$$

$$I=x-(\ln|x|+x-K)=K-\ln|x|$$
$$K=\int\frac{S}{x}dx$$

$$S=xt+1;\ x\neq 0$$
$$x^2+x+1=(xt+1)^2=x^2t^2+2xt+1$$
$$x+1=xt^2+2t$$
$$x(1-t^2)=2t-1$$
$$x=\frac{2t-1}{1-t^2};\ S=xt+1$$

$$\frac{S}{x}=\frac{xt+1}{x}=t+\frac{1}{x}=t+\frac{1-t^2}{2t-1}=\frac{t^2-t+1}{2t-1}$$
$$dx=(\frac{2t-1}{1-t^2})'dt=\frac{2(t^2-t+1)}{(t^2-1)^2}dt$$

$$K=\int\frac{S}{x}dx=\int\frac{t^2-t+1}{2t-1}\cdot\frac{2(t^2-t+1)}{(t^2-1)^2}dt=\int\frac{2(t^2-t+1)^2}{(2t-1)(t^2-1)^2}dt$$

$$\frac{2(t^2-t+1)^2}{(2t-1)(t^2-1)^2}=\frac{2}{2t-1}+\frac{1}{2(t+1)}-\frac{3}{2(t+1)^2}-\frac{1}{2(t-1)}+\frac{1}{2(t-1)^2}$$

$$K=\int(\frac{2}{2t-1}+\frac{1}{2(t+1)}-\frac{3}{2(t+1)^2}-\frac{1}{2(t-1)}+\frac{1}{2(t-1)^2})dt$$
$$K=\ln|2t-1|+\frac{1}{2}\ln|t+1|-\frac{1}{2}\ln|t-1|+\frac{3}{2(t+1)}-\frac{1}{2(t-1)}+C$$

$$t=\frac{S-1}{x}=\frac{\sqrt{1+x+x^2}-1}{x}$$
$$2t-1=\frac{2S-x-2}{x}$$
$$t+1=\frac{S+x-1}{x}$$
$$t-1=\frac{S-x-1}{x}$$

$$I=K-\ln|x|$$
$$I=\ln|\frac{2S-x-2}{x}|+\frac{1}{2}\ln|\frac{S+x-1}{S-x-1}|+\frac{3}{2}\cdot\frac{x}{S+x-1}-\frac{1}{2}\cdot\frac{x}{S-x-1}-\ln|x|+C$$

$$I=\ln|\frac{2\sqrt{1+x+x^2}-x-2}{x^2}|+\frac{1}{2}\ln|\frac{\sqrt{1+x+x^2}+x-1}{\sqrt{1+x+x^2}-x-1}|+\frac{3x}{2(\sqrt{1+x+x^2}+x-1)}-\frac{\ }{2(\sqrt{1+x+\ldots}}$$

75 номер – Д 1975

Пример:

$$\int \frac{\sqrt{x(x+1)}}{\sqrt{x}+\sqrt{x+1}}\,dx$$

Решение:

$$I = \int \frac{\sqrt{x(x+1)}}{\sqrt{x}+\sqrt{x+1}} dx = \int \frac{\sqrt{x}\sqrt{x+1}}{\sqrt{x}+\sqrt{x+1}} dx$$

$$I = \int \frac{\sqrt{x}\sqrt{x+1}}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} dx = \int \frac{\sqrt{x}\sqrt{x+1}(\sqrt{x+1}-\sqrt{x})}{(\sqrt{x+1})^2-(\sqrt{x})^2} dx$$
$$I = \int \sqrt{x}\sqrt{x+1}(\sqrt{x+1}-\sqrt{x}) dx$$

$$I = \int(\sqrt{x}(x+1) - x\sqrt{x+1}) dx = \int (x+1)\sqrt{x}\,dx - \int x\sqrt{x+1}\,dx$$
$$I = I_1 - I_2$$

$$I_1 = \int (x+1)\sqrt{x}\,dx = \int (x^{\frac{3}{2}} + x^{\frac{1}{2}}) dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

$$I_2 = \int x\sqrt{x+1}\,dx$$
$$t = x+1;\ x = t-1;\ dt = dx$$
$$I_2 = \int (t-1)\sqrt{t}\,dt = \int (t^{\frac{3}{2}} - t^{\frac{1}{2}}) dt = \frac{2}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

$$I = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}} + C$$

76 номер – Д 2025

Пример:

$$\int \frac{dx}{2\sin x - \cos x + 5}$$

Решение:

$$I = \int \frac{dx}{2\sin x - \cos x + 5}$$

$$t = \tan\frac{x}{2};\ \sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2};\ dx = \frac{2dt}{1+t^2}$$

$$I = \int \frac{\frac{2dt}{1+t^2}}{2\cdot\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} + 5} = \int \frac{2dt}{4t-(1-t^2)+5(1+t^2)} = \int \frac{2dt}{6t^2+4t+4} = \int \frac{dt}{3t^2+2t+2}$$

$$3t^2+2t+2 = 3(t+\frac{1}{3})^2 + \frac{5}{3}$$
$$I = \int \frac{dt}{3(t+\frac{1}{3})^2+\frac{5}{3}} = 3\int \frac{dt}{9(t+\frac{1}{3})^2+5}$$

$u = 3t + 1;\ du = 3dt$

$I = \int \frac{du}{u^2+5} = \frac{1}{\sqrt{5}}\arctan(\frac{u}{\sqrt{5}}) + C = \frac{1}{\sqrt{5}}\arctan(\frac{3\tan\frac{x}{2}+1}{\sqrt{5}}) + C$

$$I = \frac{1}{\sqrt{5}}\arctan(\frac{3\tan\frac{x}{2}+1}{\sqrt{5}}) + C$$

77 номер – Д 2026

Пример:

$$\int \frac{dx}{(2+\cos x)\sin x}$$

Решение:

$$I = \int \frac{dx}{(2+\cos x)\sin x}$$

$$t = \cos x;\ dt = -(\sin x)dx;\ dx = -\frac{dt}{\sin x};\ \ \sin^2 x = 1 - \cos^2 x = 1 - t^2$$

$$I = \int \frac{-\frac{dt}{\sin x}}{(2+t)\sin x} = -\int \frac{dt}{(t+2)\sin^2 x} = -\int \frac{dt}{(t+2)(1-t^2)}$$

$1 - t^2 = (1-t)(1+t)$

$I = -\int \frac{dt}{(t+2)(1-t)(1+t)}$

$$\frac{1}{(t+2)(1-t)(1+t)} = \frac{A}{t+2} + \frac{B}{1-t} + \frac{C}{1+t}$$

$1 = A(1-t)(1+t) + B(t+2)(1+t) + C(t+2)(1-t)$

$A = -\frac{1}{3};\ B = \frac{1}{6};\ C = \frac{1}{2}$

$I = -\int(-\frac{1}{3(t+2)} + \frac{1}{6(1-t)} + \frac{1}{2(1+t)})dt$

$I = \int(\frac{1}{3(t+2)} - \frac{1}{6(1-t)} - \frac{1}{2(1+t)})dt$

$$I = \frac{1}{3}\ln|t+2| + \frac{1}{6}\ln|1-t| - \frac{1}{2}\ln|1+t| + C$$

$t = \cos x$

$I = \frac{1}{3}\ln|2+\cos x| + \frac{1}{6}\ln|1-\cos x| - \frac{1}{2}\ln|1+\cos x| + C$

78 номер – Д 2027

Пример:

$\int \frac{\sin^2 x}{\sin x + 2\cos x}\, dx$

Решение:

$$I = \int \frac{\sin^2 x}{\sin x + 2\cos x} dx$$

$$\sin^2 x = (\sin x + 2\cos x)(\sin x - 2\cos x) + 4\cos^2 x$$
$$\frac{\sin^2 x}{\sin x + 2\cos x} = \sin x - 2\cos x + 4 \cdot \frac{\cos^2 x}{\sin x + 2\cos x}$$

$$I = \int (\sin x - 2\cos x) dx + 4\int \frac{\cos^2 x}{\sin x + 2\cos x} dx$$
$$I = -\cos x - 2\sin x + 4J$$

$$J = \int \frac{\cos^2 x}{\sin x + 2\cos x} dx$$
$$t = \tan\frac{x}{2};\ \sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2};\ dx = \frac{2dt}{1+t^2}$$

$$J = \int \frac{(\frac{1-t^2}{1+t^2})^2}{\frac{2t}{1+t^2} + 2\cdot\frac{1-t^2}{1+t^2}} \cdot \frac{2dt}{1+t^2} = \int \frac{(1-t^2)^2}{(1+t^2)^2(1+t-t^2)} dt$$

$$\frac{(1-t^2)^2}{(1+t^2)^2(1+t-t^2)} = -\frac{4(t-2)}{5(t^2+1)^2} - \frac{4}{5(t^2+1)} - \frac{1}{5(t^2-t-1)}$$

$$J = -\frac{4}{5}\int \frac{t-2}{(t^2+1)^2} dt - \frac{4}{5}\int \frac{dt}{t^2+1} - \frac{1}{5}\int \frac{dt}{t^2-t-1}$$

$$\int \frac{t-2}{(t^2+1)^2} dt = \int \frac{t}{(t^2+1)^2} dt - 2\int \frac{dt}{(t^2+1)^2}$$
$$u = t^2+1;\ du = 2t\, dt$$
$$\int \frac{t}{(t^2+1)^2} dt = \frac{1}{2}\int \frac{du}{u^2} = -\frac{1}{2(t^2+1)}$$

$$(\frac{t}{t^2+1})' = \frac{1-t^2}{(t^2+1)^2}$$
$$2\int \frac{dt}{(t^2+1)^2} = \frac{t}{t^2+1} + \int \frac{dt}{t^2+1} = \frac{t}{t^2+1} + \arctan t$$
$$\int \frac{dt}{(t^2+1)^2} = \frac{1}{2}(\frac{t}{t^2+1} + \arctan t)$$

$$\int \frac{t-2}{(t^2+1)^2} dt = -\frac{2t+1}{2(t^2+1)} - \arctan t$$

$t^2 - t - 1 = (t - \frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2$

$\int \frac{dt}{t^2 - t - 1} = \frac{1}{\sqrt{5}} \ln|\frac{t - \frac{1}{2} - \frac{\sqrt{5}}{2}}{t - \frac{1}{2} + \frac{\sqrt{5}}{2}}|$

$J = -\frac{4}{5}(-\frac{2t+1}{2(t^2+1)} - \arctan t) - \frac{4}{5}\arctan t - \frac{1}{5\sqrt{5}}\ln|\frac{t - \frac{1}{2} - \frac{\sqrt{5}}{2}}{t - \frac{1}{2} + \frac{\sqrt{5}}{2}}|$

$J = \frac{4t+2}{5(t^2+1)} + \frac{\sqrt{5}}{25}\ln|\frac{2t - 1 + \sqrt{5}}{2t - 1 - \sqrt{5}}| + C$

$t = \tan\frac{x}{2}$

$I = -\frac{\cos x + 2\sin x}{5} + \frac{4\sqrt{5}}{25}\ln|\frac{2\tan\frac{x}{2} - 1 + \sqrt{5}}{2\tan\frac{x}{2} - 1 - \sqrt{5}}| + C$

79 номер – Д 2029

Пример:

$\int \frac{\sin^2 x}{1 + \sin^2 x}\, dx$

Решение:

$I = \int \frac{\sin^2 x}{1 + \sin^2 x} dx$

$\frac{\sin^2 x}{1 + \sin^2 x} = 1 - \frac{1}{1 + \sin^2 x}$

$I = \int dx - \int \frac{dx}{1 + \sin^2 x} = x - J$

$J = \int \frac{dx}{1 + \sin^2 x}$

$t = \tan x;\ dt = (1 + t^2)dx;\ dx = \frac{dt}{1 + t^2}$

$\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x} = \frac{t^2}{1 + t^2}$

$J = \int \frac{\frac{dt}{1+t^2}}{1 + \frac{t^2}{1+t^2}} = \int \frac{\frac{dt}{1+t^2}}{\frac{1+2t^2}{1+t^2}} = \int \frac{dt}{1 + 2t^2}$

$J = \frac{1}{\sqrt{2}}\arctan(\sqrt{2}\,t) + C = \frac{1}{\sqrt{2}}\arctan(\sqrt{2}\tan x) + C$

$I = x - \frac{1}{\sqrt{2}}\arctan(\sqrt{2}\tan x) + C$

80 номер – Д 2030

Пример:

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

Решение:

$$I = \int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

$$I = \int \frac{dx}{\cos^2 x(a^2 \tan^2 x + b^2)} = \int \frac{\sec^2 x}{a^2 \tan^2 x + b^2} dx$$

$t = \tan x;\ dt = \sec^2 x\, dx$

$$I = \int \frac{dt}{a^2 t^2 + b^2} = \frac{1}{b^2} \int \frac{dt}{1 + (\frac{a}{b})^2 t^2}$$

$$I = \frac{1}{b^2} \cdot \frac{b}{a} \arctan(\frac{a}{b} t) + C = \frac{1}{ab} \arctan(\frac{a}{b} \tan x) + C$$

$$I = \frac{1}{ab} \arctan(\frac{a \tan x}{b}) + C$$

81 номер – Д 2031

Пример:

$$\int \frac{\cos^2 x dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2}$$

Решение:

$$I = \int \frac{\cos^2 x}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} dx;\ (a \neq 0,\ b \neq 0)$$

$$t = \tan x;\ dt = (\tan x)' dx = \sec^2 x\, dx = (1 + \tan^2 x) dx = (1 + t^2) dx;\ dx = \frac{dt}{1 + t^2};$$

$$\sin^2 x = \frac{t^2}{1 + t^2};\ \cos^2 x = \frac{1}{1 + t^2};$$

$$I = \int \frac{\frac{1}{1 + t^2} \cdot \frac{dt}{1 + t^2}}{(a^2 \frac{t^2}{1 + t^2} + b^2 \frac{1}{1 + t^2})^2} = \int \frac{\frac{dt}{(1 + t^2)^2}}{(\frac{a^2 t^2 + b^2}{1 + t^2})^2} = \int \frac{dt}{(a^2 t^2 + b^2)^2}$$

$u = \frac{a}{b} t;\ t = \frac{b}{a} u;\ dt = \frac{b}{a} du;$

$a^2 t^2 + b^2 = b^2 (u^2 + 1);$

$$I = \int \frac{\frac{b}{a}du}{(b^2(u^2+1))^2} = \frac{1}{ab^3}\int \frac{du}{(u^2+1)^2}$$

$$(\frac{u}{1+u^2})' = \frac{1-u^2}{(1+u^2)^2}$$
$$\frac{1}{(1+u^2)^2} = \frac{1}{2}(\frac{1-u^2}{(1+u^2)^2} + \frac{1}{1+u^2})$$

$$\int \frac{du}{(1+u^2)^2} = \frac{1}{2}\int \frac{1-u^2}{(1+u^2)^2}du + \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\cdot\frac{u}{1+u^2} + \frac{1}{2}\arctan u$$

$$I = \frac{1}{ab^3}(\frac{1}{2}\cdot\frac{u}{1+u^2} + \frac{1}{2}\arctan u) = \frac{1}{2ab^3}(\frac{u}{1+u^2} + \arctan u) + C$$

$$u = \frac{a}{b}\tan x;$$
$$\frac{u}{1+u^2} = \frac{\frac{a}{b}\tan x}{1+(\frac{a}{b}\tan x)^2} = \frac{ab\tan x}{b^2+a^2\tan^2 x} = \frac{ab\sin x\cos x}{a^2\sin^2 x + b^2\cos^2 x}$$

$$I = \frac{\sin x\cos x}{2b^2(a^2\sin^2 x + b^2\cos^2 x)} + \frac{1}{2ab^3}\arctan(\frac{a\tan x}{b}) + C$$

82 номер – Д 2032

Пример:

$$\int \frac{\sin x\cos x}{\sin x + \cos x}\,dx$$

Решение:

$$\sin x + \cos x \neq 0;$$

$$I = \int \frac{\sin x\cos x}{\sin x + \cos x}dx$$

$$t = x - \frac{\pi}{4};\ x = t + \frac{\pi}{4};\ dt = dx$$
$$\sin x = \sin(t+\frac{\pi}{4}) = \frac{\sin t + \cos t}{\sqrt{2}};$$
$$\cos x = \cos(t+\frac{\pi}{4}) = \frac{\cos t - \sin t}{\sqrt{2}};$$

$$\sin x + \cos x = \frac{\sin t + \cos t + \cos t - \sin t}{\sqrt{2}} = \sqrt{2}\cos t;$$
$$\sin x\cos x = \frac{(\sin t + \cos t)(\cos t - \sin t)}{2} = \frac{\cos^2 t - \sin^2 t}{2} = \frac{1}{2}\cos 2t;$$

$$I = \int \frac{\frac{1}{2}\cos 2t}{\sqrt{2}\cos t}dt = \frac{1}{2\sqrt{2}}\int \frac{\cos 2t}{\cos t}dt$$
$$\cos 2t = 2\cos^2 t - 1$$

$$\frac{\cos 2t}{\cos t} = 2\cos t - \sec t$$

$$I = \frac{1}{2\sqrt{2}}\int(2\cos t - \sec t)dt = \frac{1}{\sqrt{2}}\int \cos t\, dt - \frac{1}{2\sqrt{2}}\int \sec t\, dt$$

$$I = \frac{1}{\sqrt{2}}\sin t - \frac{1}{2\sqrt{2}}\ln|\sec t + \tan t| + C$$

$$\sin t = \sin(x - \frac{\pi}{4}) = \frac{\sin x - \cos x}{\sqrt{2}};$$

$$\sec t + \tan t = \frac{1 + \sin t}{\cos t};\ \sin t = \frac{\sin x - \cos x}{\sqrt{2}};\ \cos t = \cos(x - \frac{\pi}{4}) = \frac{\sin x + \cos x}{\sqrt{2}};$$

$$\sec t + \tan t = \frac{\sqrt{2} + \sin x - \cos x}{\sin x + \cos x}$$

$$I = \frac{\sin x - \cos x}{2} - \frac{1}{2\sqrt{2}}\ln|\frac{\sqrt{2} + \sin x - \cos x}{\sin x + \cos x}| + C$$

83 номер – Д 2033

Пример:

$$\int \frac{dx}{(a\sin x + b\cos x)^2}$$

Решение:

$$a^2 + b^2 \neq 0;\ a\sin x + b\cos x \neq 0;$$

$$I = \int \frac{dx}{(a\sin x + b\cos x)^2}$$

$$u = a\sin x + b\cos x;\ u' = a\cos x - b\sin x$$
$$v = a\cos x - b\sin x;\ v' = -a\sin x - b\cos x = -u$$

$$(\frac{v}{u})' = \frac{v'u - vu'}{u^2} = \frac{(-u)u - v^2}{u^2} = -\frac{u^2 + v^2}{u^2}$$

$$u^2 + v^2 = (a\sin x + b\cos x)^2 + (a\cos x - b\sin x)^2 = a^2 + b^2$$

$$(\frac{v}{u})' = -\frac{a^2 + b^2}{(a\sin x + b\cos x)^2}$$

$$\frac{1}{(a\sin x + b\cos x)^2} = -\frac{1}{a^2 + b^2}(\frac{v}{u})'$$

$$I = -\frac{1}{a^2 + b^2}\int(\frac{v}{u})'dx = -\frac{1}{a^2 + b^2}\cdot\frac{v}{u} + C$$

$$I = -\frac{a\cos x - b\sin x}{(a^2 + b^2)(a\sin x + b\cos x)} + C$$

84 номер – Д 2072

Пример:

$\int x^7 e^{-x^2}\, dx$

Решение:

$I = \int x^7 e^{-x^2} dx$

$t = x^2;\ dt = 2x\, dx;\ x^7 dx = x^6 \cdot x\, dx = (x^2)^3 \cdot x\, dx = t^3 \cdot \frac{1}{2} dt$
$I = \frac{1}{2} \int t^3 e^{-t} dt$

$J = \int t^3 e^{-t} dt;\ J = \int u\, dv;\ u = t^3;\ dv = e^{-t} dt$
$du = 3t^2 dt;\ v = -e^{-t}$
$J = uv - \int v\, du = -t^3 e^{-t} + 3 \int t^2 e^{-t} dt$

$J_1 = \int t^2 e^{-t} dt;\ J_1 = \int u\, dv;\ u = t^2;\ dv = e^{-t} dt$
$du = 2t\, dt;\ v = -e^{-t}$
$J_1 = -t^2 e^{-t} + 2 \int t e^{-t} dt$

$J_2 = \int t e^{-t} dt;\ J_2 = \int u\, dv;\ u = t;\ dv = e^{-t} dt$
$du = dt;\ v = -e^{-t}$
$J_2 = -t e^{-t} + \int e^{-t} dt = -t e^{-t} - e^{-t}$

$J_1 = -t^2 e^{-t} + 2(-t e^{-t} - e^{-t}) = -(t^2 + 2t + 2) e^{-t}$
$J = -t^3 e^{-t} + 3J_1 = -t^3 e^{-t} - 3(t^2 + 2t + 2) e^{-t} = -(t^3 + 3t^2 + 6t + 6) e^{-t}$

$I = -\frac{1}{2}(t^3 + 3t^2 + 6t + 6) e^{-t} + C$
$t = x^2$

$I = -\frac{1}{2} e^{-x^2} (x^6 + 3x^4 + 6x^2 + 6) + C$

85 номер – Д 2073

Пример:

$\int x^2 e^{\sqrt{x}} dx$

Решение:

$x \geq 0$
$I = \int x^2 e^{\sqrt{x}} dx$

$t = \sqrt{x};\ x = t^2;\ dx = 2t\, dt$
$I = \int t^4 e^t \cdot 2t\, dt = 2 \int t^5 e^t dt$
$I = 2J$

$J = \int t^5 e^t dt;\ J = \int u\, dv;\ u = t^5;\ dv = e^t dt$
$du = 5t^4 dt;\ v = e^t$
$J = t^5 e^t - 5 \int t^4 e^t dt = t^5 e^t - 5J_1$

$J_1 = \int t^4 e^t dt = t^4 e^t - 4\int t^3 e^t dt = t^4 e^t - 4J_2$
$J_2 = \int t^3 e^t dt = t^3 e^t - 3\int t^2 e^t dt = t^3 e^t - 3J_3$
$J_3 = \int t^2 e^t dt = t^2 e^t - 2\int t e^t dt = t^2 e^t - 2J_4$
$J_4 = \int t e^t dt = t e^t - \int e^t dt = t e^t - e^t$

$J_3 = t^2 e^t - 2(te^t - e^t) = e^t(t^2 - 2t + 2)$
$J_2 = t^3 e^t - 3e^t(t^2 - 2t + 2) = e^t(t^3 - 3t^2 + 6t - 6)$
$J_1 = t^4 e^t - 4e^t(t^3 - 3t^2 + 6t - 6) = e^t(t^4 - 4t^3 + 12t^2 - 24t + 24)$
$J = t^5 e^t - 5e^t(t^4 - 4t^3 + 12t^2 - 24t + 24) = e^t(t^5 - 5t^4 + 20t^3 - 60t^2 + 120t - 120)$

$I = 2e^t(t^5 - 5t^4 + 20t^3 - 60t^2 + 120t - 120) + C$
$t = \sqrt{x}$

$I = 2e^{\sqrt{x}}(x^2\sqrt{x} - 5x^2 + 20x\sqrt{x} - 60x + 120\sqrt{x} - 120) + C$

86 номер – Д 2074

Пример:

$\int e^{ax}\cos^2 bx\,dx$

Решение:

$I = \int e^{ax}\cos^2(bx)\,dx$

$\cos^2(bx) = \dfrac{1 + \cos(2bx)}{2}$
$I = \dfrac{1}{2}\int e^{ax}dx + \dfrac{1}{2}\int e^{ax}\cos(2bx)\,dx$
$I = I_1 + I_2$

$a \neq 0;$
$I_1 = \dfrac{1}{2}\int e^{ax}dx = \dfrac{1}{2}\cdot\dfrac{e^{ax}}{a} = \dfrac{e^{ax}}{2a}$

$I_2 = \dfrac{1}{2}\int e^{ax}\cos(2bx)\,dx$
$\int e^{ax}\cos(kx)\,dx = \dfrac{e^{ax}}{a^2 + k^2}(a\cos(kx) + k\sin(kx))$
$k = 2b$
$I_2 = \dfrac{1}{2}\cdot\dfrac{e^{ax}}{a^2 + 4b^2}(a\cos(2bx) + 2b\sin(2bx))$

$I = \dfrac{e^{ax}}{2a} + \dfrac{e^{ax}}{2(a^2 + 4b^2)}(a\cos(2bx) + 2b\sin(2bx)) + C$

$a = 0;$
$I = \int\cos^2(bx)\,dx = \int\dfrac{1 + \cos(2bx)}{2}dx = \dfrac{x}{2} + \dfrac{\sin(2bx)}{4b} + C;\ (b \neq 0)$

87 номер – Д 2075

Пример:

$\int e^{ax}\sin^3 bx\,dx$

Решение:

$I = \int e^{ax}\sin^3(bx)\,dx$

$$\sin 3u = 3\sin u - 4\sin^3 u$$
$$\sin^3 u = \frac{3\sin u - \sin 3u}{4}$$
$$u = bx$$
$$\sin^3(bx) = \frac{3\sin(bx) - \sin(3bx)}{4}$$

$$I = \frac{1}{4}\int e^{ax}(3\sin(bx) - \sin(3bx))dx = \frac{3}{4}\int e^{ax}\sin(bx)dx - \frac{1}{4}\int e^{ax}\sin(3bx)dx$$
$$I = \frac{3}{4}I_1 - \frac{1}{4}I_2$$

$$\int e^{ax}\sin(kx)\,dx = \frac{e^{ax}}{a^2+k^2}(a\sin(kx) - k\cos(kx)) + C$$

$$I_1 = \int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin(bx) - b\cos(bx))$$
$$I_2 = \int e^{ax}\sin(3bx)\,dx = \frac{e^{ax}}{a^2+9b^2}(a\sin(3bx) - 3b\cos(3bx))$$

$$I = \frac{e^{ax}}{4}\left(\frac{3(a\sin bx - b\cos bx)}{a^2+b^2} - \frac{a\sin 3bx - 3b\cos 3bx}{a^2+9b^2}\right) + C$$

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Пример:

$\int xe^x\sin x\,dx$

Решение:

$I = \int xe^x\sin x\,dx$

$$I = \int u\,dv;\ \int u\,dv = uv - \int v\,du$$
$$u = x;\ du = dx$$
$$dv = e^x\sin x\,dx$$

$$v = \int e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x)$$

$$I = x\cdot\frac{e^x}{2}(\sin x - \cos x) - \int\frac{e^x}{2}(\sin x - \cos x)\,dx$$
$$I = \frac{xe^x}{2}(\sin x - \cos x) - \frac{1}{2}\int e^x\sin x\,dx + \frac{1}{2}\int e^x\cos x\,dx$$

$$\int e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x)$$
$$\int e^x\cos x\,dx = \frac{e^x}{2}(\sin x + \cos x)$$

$$I = \frac{xe^x}{2}(\sin x - \cos x) - \frac{1}{2} \cdot \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2} \cdot \frac{e^x}{2}(\sin x + \cos x) + C$$
$$I = \frac{xe^x}{2}(\sin x - \cos x) + \frac{e^x}{2}\cos x + C$$

$$I = \frac{e^x}{2}(x\sin x + (1 - x)\cos x) + C$$

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Пример:

$\int x^2 e^x \cos x \, dx$

Решение:

$$I = \int x^2 e^x \cos x \, dx$$

$$I = \int u \, dv; \;\; \int u \, dv = uv - \int v \, du$$
$$u = x^2; \; du = 2x \, dx$$
$$dv = e^x \cos x \, dx$$

$$v = \int e^x \cos x \, dx = \frac{e^x}{2}(\sin x + \cos x)$$

$$I = \frac{x^2 e^x}{2}(\sin x + \cos x) - \int xe^x(\sin x + \cos x) \, dx$$
$$I = \frac{x^2 e^x}{2}(\sin x + \cos x) - J$$

$$J = \int xe^x(\sin x + \cos x) \, dx = \int xe^x \sin x \, dx + \int xe^x \cos x \, dx$$
$$J = J_1 + J_2$$

$$J_1 = \int xe^x \sin x \, dx$$
$$J_1 = \int u \, dv; \; u = x; \; du = dx; \; dv = e^x \sin x \, dx$$
$$\int e^x \sin x \, dx = \frac{e^x}{2}(\sin x - \cos x)$$
$$J_1 = x \cdot \frac{e^x}{2}(\sin x - \cos x) - \int \frac{e^x}{2}(\sin x - \cos x) \, dx$$
$$\int e^x \sin x \, dx = \frac{e^x}{2}(\sin x - \cos x); \;\; \int e^x \cos x \, dx = \frac{e^x}{2}(\sin x + \cos x)$$
$$J_1 = \frac{xe^x}{2}(\sin x - \cos x) - \frac{1}{2} \cdot \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2} \cdot \frac{e^x}{2}(\sin x + \cos x)$$
$$J_1 = \frac{e^x}{2}(x\sin x + (1 - x)\cos x)$$

$$J_2 = \int xe^x \cos x \, dx$$
$$J_2 = \int u \, dv; \; u = x; \; du = dx; \; dv = e^x \cos x \, dx$$
$$J_2 = x \cdot \frac{e^x}{2}(\sin x + \cos x) - \int \frac{e^x}{2}(\sin x + \cos x) \, dx$$
$$\int e^x(\sin x + \cos x) \, dx = \int e^x \sin x \, dx + \int e^x \cos x \, dx = e^x \sin x$$
$$J_2 = \frac{xe^x}{2}(\sin x + \cos x) - \frac{e^x}{2}\sin x$$

$$J = \frac{e^x}{2}(x\sin x + (1-x)\cos x) + \frac{e^x}{2}(x(\sin x + \cos x) - \sin x) = \frac{e^x}{2}((2x-1)\sin x + \cos x)$$

$$I = \frac{x^2 e^x}{2}(\sin x + \cos x) - \frac{e^x}{2}((2x-1)\sin x + \cos x) + C$$

$$I = \frac{e^x}{2}((x-1)^2 \sin x + (x^2-1)\cos x) + C$$