30 номер

Пример:

$x^3 + 1 = 0$

Решение:

$x^3 + 1 = (x + 1)(x^2 - x + 1)$
$x = -1;$
$x^2 - x + 1 = 0$
$D = -3$

Ответ: $x = \frac{1 \pm i\sqrt{3}}{2}$

31 номер

Пример:

$x^4 - 4x^2 + 5 = 0$

Решение:

$t = x^2$
$t^2 - 4t + 5 = 0$
$D = -4$
$t = \frac{4 \pm 2i}{2} = 2 \pm i$

$x^2 = 2 \pm i$

1. $x^2 = 2 + i$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = 2 + i$

$$\begin{cases} a^2 - b^2 = 2 \\ 2abi = i; 2ab = 1 \end{cases}$$

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 = 2^2 + 1^2 = 5$
$a^2 + b^2 = \sqrt{5}$
$a^2 = \frac{(a^2 + b^2) + (a^2 - b^2)}{2} = \frac{\sqrt{5} + 2}{2}$
$b^2 = \frac{(a^2 + b^2) - (a^2 - b^2)}{2} = \frac{\sqrt{5} - 2}{2}$
$x_1 = \sqrt{\frac{\sqrt{5} + 2}{2}} + i\sqrt{\frac{\sqrt{5} - 2}{2}}$
$x_2 = -(\sqrt{\frac{\sqrt{5} + 2}{2}} + i\sqrt{\frac{\sqrt{5} - 2}{2}})$

2. $x^2 = 2 - i$
$2 - i = \overline{(2 + i)}$
$x_3 = \sqrt{\frac{\sqrt{5} + 2}{2}} - i\sqrt{\frac{\sqrt{5} - 2}{2}}$
$x_4 = -(\sqrt{\frac{\sqrt{5} + 2}{2}} - i\sqrt{\frac{\sqrt{5} - 2}{2}})$

Ответ:

$$x_1 = \sqrt{\frac{\sqrt{5} + 2}{2}} + i\sqrt{\frac{\sqrt{5} - 2}{2}}$$
$$x_2 = -(\sqrt{\frac{\sqrt{5} + 2}{2}} + i\sqrt{\frac{\sqrt{5} - 2}{2}})$$
$$x_3 = \sqrt{\frac{\sqrt{5} + 2}{2}} - i\sqrt{\frac{\sqrt{5} - 2}{2}}$$
$$x_4 = -(\sqrt{\frac{\sqrt{5} + 2}{2}} - i\sqrt{\frac{\sqrt{5} - 2}{2}})$$

32 номер

Пример:

$x^4 + 4x^2 + 20 = 0$

Решение:

$t = x^2$
$t^2 + 4t + 20 = 0$
$D = -64$
$t = \frac{-4 \pm 8i}{2} = -2 \pm 4i$

$x^2 = -2 \pm 4i$

1. $x^2 = -2 + 4i$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = -2 + 4i$

$$\begin{cases} a^2 - b^2 = -2 \\ 2ab = 4 \end{cases}$$

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 = (-2)^2 + 4^2 = 20$
$a^2 + b^2 = \sqrt{20} = 2\sqrt{5}$
$a^2 = \frac{(a^2 + b^2) + (a^2 - b^2)}{2} = \frac{2\sqrt{5} - 2}{2} = \sqrt{5} - 1$
$b^2 = \frac{(a^2 + b^2) - (a^2 - b^2)}{2} = \frac{2\sqrt{5} + 2}{2} = \sqrt{5} + 1$

$x_1 = \sqrt{\sqrt{5}-1} + i\sqrt{\sqrt{5}+1}$
$x_2 = -(\sqrt{\sqrt{5}-1} + i\sqrt{\sqrt{5}+1})$

2. $x^2 = -2 - 4i$
$-2 - 4i = \overline{(-2+4i)}$

$x_3 = \sqrt{\sqrt{5}-1} - i\sqrt{\sqrt{5}+1}$
$x_4 = -(\sqrt{\sqrt{5}-1} - i\sqrt{\sqrt{5}+1})$

Ответ:

$$x_1 = \sqrt{\sqrt{5}-1} + i\sqrt{\sqrt{5}+1}$$
$$x_2 = -(\sqrt{\sqrt{5}-1} + i\sqrt{\sqrt{5}+1})$$
$$x_3 = \sqrt{\sqrt{5}-1} - i\sqrt{\sqrt{5}+1}$$
$$x_4 = -(\sqrt{\sqrt{5}-1} - i\sqrt{\sqrt{5}+1})$$

33 номер

Пример:

$x^4 - 6x^2 + 13 = 0$

Решение:

$t = x^2$
$t^2 - 6t + 13 = 0$
$D = -16$
$t = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

$x^2 = 3 \pm 2i$

1. $x^2 = 3 + 2i$
$x = a + bi$
$(a+bi)^2 = (a^2 - b^2) + 2abi = 3 + 2i$

$$\begin{cases} a^2 - b^2 = 3 \\ 2ab = 2 \end{cases}$$

$(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2 = 3^2 + 2^2 = 13$
$a^2 + b^2 = \sqrt{13}$
$a^2 = \dfrac{(a^2+b^2)+(a^2-b^2)}{2} = \dfrac{\sqrt{13}+3}{2}$
$b^2 = \dfrac{(a^2+b^2)-(a^2-b^2)}{2} = \dfrac{\sqrt{13}-3}{2}$

$x_1 = \sqrt{\frac{\sqrt{13}+3}{2}} + i\sqrt{\frac{\sqrt{13}-3}{2}}$
$x_2 = -(\sqrt{\frac{\sqrt{13}+3}{2}} + i\sqrt{\frac{\sqrt{13}-3}{2}})$

2. $x^2 = 3 - 2i$
$3 - 2i = \overline{(3 + 2i)}$

$x_3 = \sqrt{\frac{\sqrt{13}+3}{2}} - i\sqrt{\frac{\sqrt{13}-3}{2}}$
$x_4 = -(\sqrt{\frac{\sqrt{13}+3}{2}} - i\sqrt{\frac{\sqrt{13}-3}{2}})$

Ответ:

$$x_1 = \sqrt{\frac{\sqrt{13}+3}{2}} + i\sqrt{\frac{\sqrt{13}-3}{2}}$$
$$x_2 = -(\sqrt{\frac{\sqrt{13}+3}{2}} + i\sqrt{\frac{\sqrt{13}-3}{2}})$$
$$x_3 = \sqrt{\frac{\sqrt{13}+3}{2}} - i\sqrt{\frac{\sqrt{13}-3}{2}}$$
$$x_4 = -(\sqrt{\frac{\sqrt{13}+3}{2}} - i\sqrt{\frac{\sqrt{13}-3}{2}})$$

34 номер

Пример:

$x^4 + 2x^2 + 17 = 0$

Решение:

$t = x^2$
$t^2 + 2t + 17 = 0$
$D = -64$
$t = \frac{-2 \pm 8i}{2} = -1 \pm 4i$

$x^2 = -1 \pm 4i$

1. $x^2 = -1 + 4i$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = -1 + 4i$

$$\begin{cases} a^2 - b^2 = -1 \\ 2ab = 4 \end{cases}$$

$(a^2+b^2)^2=(a^2-b^2)^2+(2ab)^2=(-1)^2+4^2=17$
$a^2+b^2=\sqrt{17}$
$a^2=\frac{(a^2+b^2)+(a^2-b^2)}{2}=\frac{\sqrt{17}-1}{2}$
$b^2=\frac{(a^2+b^2)-(a^2-b^2)}{2}=\frac{\sqrt{17}+1}{2}$

$x_1=\sqrt{\frac{\sqrt{17}-1}{2}}+i\sqrt{\frac{\sqrt{17}+1}{2}}$
$x_2=-(\sqrt{\frac{\sqrt{17}-1}{2}}+i\sqrt{\frac{\sqrt{17}+1}{2}})$

2. $x^2=-1-4i$
$-1-4i=\overline{(-1+4i)}$

$x_3=\sqrt{\frac{\sqrt{17}-1}{2}}-i\sqrt{\frac{\sqrt{17}+1}{2}}$
$x_4=-(\sqrt{\frac{\sqrt{17}-1}{2}}-i\sqrt{\frac{\sqrt{17}+1}{2}})$

Ответ:

$$x_1=\sqrt{\frac{\sqrt{17}-1}{2}}+i\sqrt{\frac{\sqrt{17}+1}{2}}$$
$$x_2=-(\sqrt{\frac{\sqrt{17}-1}{2}}+i\sqrt{\frac{\sqrt{17}+1}{2}})$$
$$x_3=\sqrt{\frac{\sqrt{17}-1}{2}}-i\sqrt{\frac{\sqrt{17}+1}{2}}$$
$$x_4=-(\sqrt{\frac{\sqrt{17}-1}{2}}-i\sqrt{\frac{\sqrt{17}+1}{2}})$$

35 номер

Пример:

$x^4+10x^2+61=0$

Решение:

$t=x^2$
$t^2+10t+61=0$
$D-144$
$t=\frac{-10\pm12i}{2}=-5\pm6i$

$x^2=-5\pm6i$

1. $x^2 = -5 + 6i$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = -5 + 6i$

$$\begin{cases} a^2 - b^2 = -5 \\ 2ab = 6 \end{cases}$$

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 = (-5)^2 + 6^2 = 61$
$a^2 + b^2 = \sqrt{61}$
$a^2 = \frac{(a^2 + b^2) + (a^2 - b^2)}{2} = \frac{\sqrt{61} - 5}{2}$
$b^2 = \frac{(a^2 + b^2) - (a^2 - b^2)}{2} = \frac{\sqrt{61} + 5}{2}$

$x_1 = \sqrt{\frac{\sqrt{61} - 5}{2}} + i\sqrt{\frac{\sqrt{61} + 5}{2}}$
$x_2 = -(\sqrt{\frac{\sqrt{61} - 5}{2}} + i\sqrt{\frac{\sqrt{61} + 5}{2}})$

2. $x^2 = -5 - 6i$
$-5 - 6i = \overline{(-5 + 6i)}$

$x_3 = \sqrt{\frac{\sqrt{61} - 5}{2}} - i\sqrt{\frac{\sqrt{61} + 5}{2}}$
$x_4 = -(\sqrt{\frac{\sqrt{61} - 5}{2}} - i\sqrt{\frac{\sqrt{61} + 5}{2}})$

Ответ:

$$x_1 = \sqrt{\frac{\sqrt{61} - 5}{2}} + i\sqrt{\frac{\sqrt{61} + 5}{2}}$$
$$x_2 = -(\sqrt{\frac{\sqrt{61} - 5}{2}} + i\sqrt{\frac{\sqrt{61} + 5}{2}})$$
$$x_3 = \sqrt{\frac{\sqrt{61} - 5}{2}} - i\sqrt{\frac{\sqrt{61} + 5}{2}}$$
$$x_4 = -(\sqrt{\frac{\sqrt{61} - 5}{2}} - i\sqrt{\frac{\sqrt{61} + 5}{2}})$$

36 номер

Пример:

$x^4 - x^2 + 37 = 0$

Решение:

$t = x^2$
$t^2 - t + 37 = 0$
$D - 147$
$t = \frac{1 \pm \sqrt{-147}}{2} = \frac{1 \pm 7i\sqrt{3}}{2}$

$x^2 = \frac{1 \pm 7i\sqrt{3}}{2}$

1. $x^2 = \frac{1 + 7i\sqrt{3}}{2}$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = \frac{1 + 7i\sqrt{3}}{2}$

$$\begin{cases} a^2 - b^2 = \frac{1}{2} \\ 2ab = \frac{7\sqrt{3}}{2} \end{cases}$$

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 = (\frac{1}{2})^2 + (\frac{7\sqrt{3}}{2})^2 = \frac{1}{4} + \frac{147}{4} = 37$
$a^2 + b^2 = \sqrt{37}$

$a^2 = \frac{(a^2 + b^2) + (a^2 - b^2)}{2} = \frac{\sqrt{37} + \frac{1}{2}}{2} = \frac{2\sqrt{37} + 1}{4}$

$b^2 = \frac{(a^2 + b^2) - (a^2 - b^2)}{2} = \frac{\sqrt{37} - \frac{1}{2}}{2} = \frac{2\sqrt{37} - 1}{4}$

$x_1 = \sqrt{\frac{2\sqrt{37} + 1}{4}} + i\sqrt{\frac{2\sqrt{37} - 1}{4}}$
$x_2 = -(\sqrt{\frac{2\sqrt{37} + 1}{4}} + i\sqrt{\frac{2\sqrt{37} - 1}{4}})$

2. $x^2 = \frac{1 - 7i\sqrt{3}}{2}$
$\frac{1 - 7i\sqrt{3}}{2} = \overline{(\frac{1 + 7i\sqrt{3}}{2})}$

$x_3 = \sqrt{\frac{2\sqrt{37} + 1}{4}} - i\sqrt{\frac{2\sqrt{37} - 1}{4}}$
$x_4 = -(\sqrt{\frac{2\sqrt{37} + 1}{4}} - i\sqrt{\frac{2\sqrt{37} - 1}{4}})$

Ответ:

$$x_1 = \sqrt{\frac{2\sqrt{37}+1}{4}} + i\sqrt{\frac{2\sqrt{37}-1}{4}}$$
$$x_2 = -(\sqrt{\frac{2\sqrt{37}+1}{4}} + i\sqrt{\frac{2\sqrt{37}-1}{4}})$$
$$x_3 = \sqrt{\frac{2\sqrt{37}+1}{4}} - i\sqrt{\frac{2\sqrt{37}-1}{4}}$$
$$x_4 = -(\sqrt{\frac{2\sqrt{37}+1}{4}} - i\sqrt{\frac{2\sqrt{37}-1}{4}})$$

37 номер

Пример:

$x^4 + 6x^2 + 8 = 0$

Решение:

$t = x^2$
$t^2 + 6t + 8 = 0$
$D = 6^2 - 4 \cdot 1 \cdot 8 = 36 - 32 = 4$
$t = \frac{-6 \pm \sqrt{4}}{2} = -3 \pm 1$

$t_1 = -2, \quad t_2 = -4$

$x^2 = -2 \text{ or } x^2 = -4$

1. $x^2 = -2$
$x = \pm\sqrt{-2} = \pm i\sqrt{2}$
$x_1 = i\sqrt{2}$
$x_2 = -i\sqrt{2}$

2. $x^2 = -4$
$x = \pm\sqrt{-4} = \pm 2i$
$x_3 = 2i$
$x_4 = -2i$

Ответ:

$$x_1 = i\sqrt{2}$$
$$x_2 = -i\sqrt{2}$$
$$x_3 = 2i$$
$$x_4 = -2i$$

38 номер

Пример:

$x4 + 8x^2 + 41 = 0$

Решение:

$t = x^2$
$t^2 + 8t + 41 = 0$
$D = 8^2 - 4 \cdot 1 \cdot 41 = 64 - 164 = -100$
$t = \frac{-8 \pm \sqrt{-100}}{2} = \frac{-8 \pm 10i}{2} = -4 \pm 5i$

$x^2 = -4 \pm 5i$

1. $x^2 = -4 + 5i$
$x = a + bi$
$(a + bi)^2 = (a^2 - b^2) + 2abi = -4 + 5i$

$$\begin{cases} a^2 - b^2 = -4 \\ 2ab = 5 \end{cases}$$

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2 = (-4)^2 + 5^2 = 41$
$a^2 + b^2 = \sqrt{41}$
$a^2 = \frac{(a^2 + b^2) + (a^2 - b^2)}{2} = \frac{\sqrt{41} - 4}{2}$
$b^2 = \frac{(a^2 + b^2) - (a^2 - b^2)}{2} = \frac{\sqrt{41} + 4}{2}$

$x_1 = \sqrt{\frac{\sqrt{41} - 4}{2}} + i\sqrt{\frac{\sqrt{41} + 4}{2}}$
$x_2 = -(\sqrt{\frac{\sqrt{41} - 4}{2}} + i\sqrt{\frac{\sqrt{41} + 4}{2}})$

2. $x^2 = -4 - 5i$
$-4 - 5i = \overline{(-4 + 5i)}$

$x_3 = \sqrt{\frac{\sqrt{41} - 4}{2}} - i\sqrt{\frac{\sqrt{41} + 4}{2}}$
$x_4 = -(\sqrt{\frac{\sqrt{41} - 4}{2}} - i\sqrt{\frac{\sqrt{41} + 4}{2}})$

Ответ:

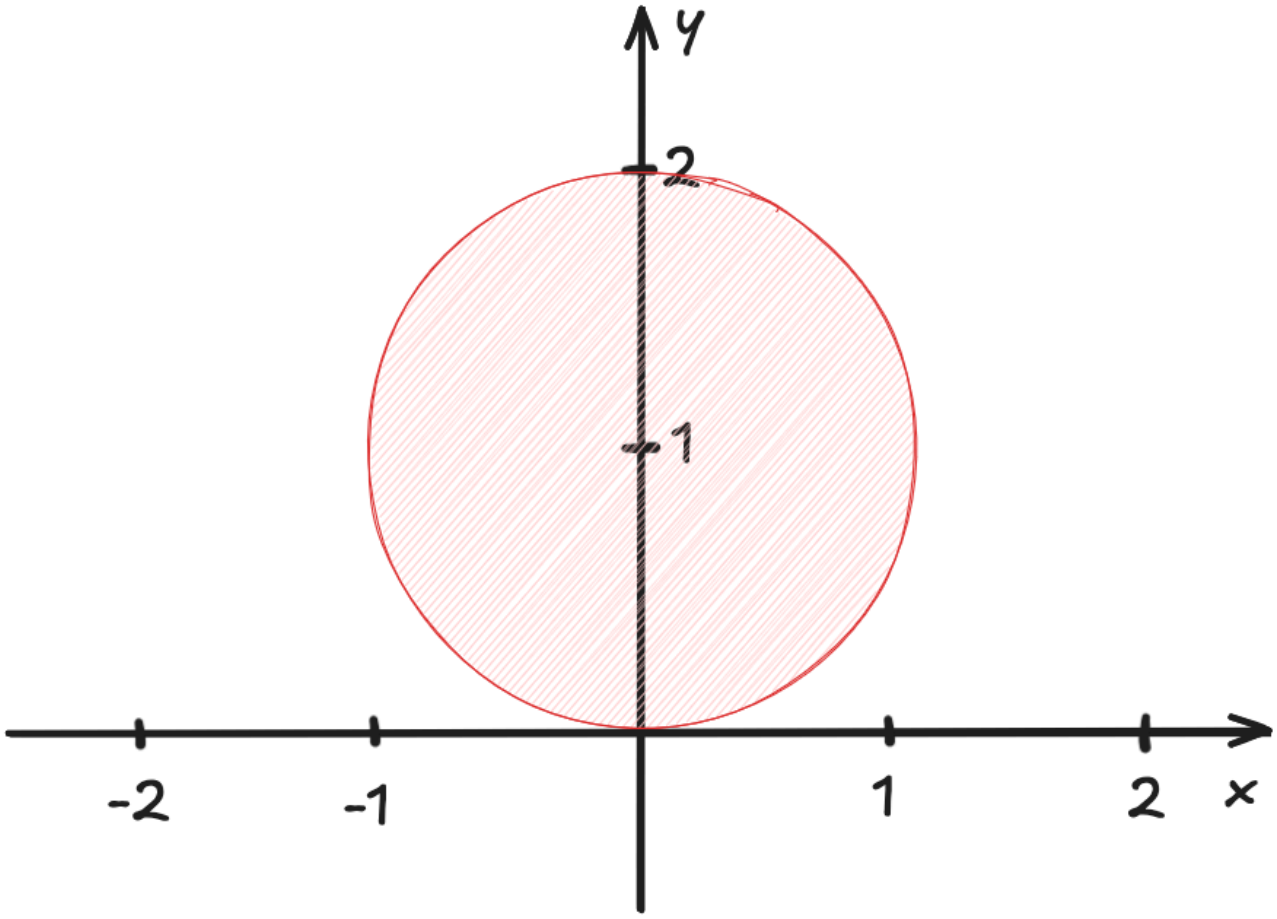
$$x_1 = \sqrt{\frac{\sqrt{41} - 4}{2}} + i\sqrt{\frac{\sqrt{41} + 4}{2}}$$
$$x_2 = -(\sqrt{\frac{\sqrt{41} - 4}{2}} + i\sqrt{\frac{\sqrt{41} + 4}{2}})$$
$$x_3 = \sqrt{\frac{\sqrt{41} - 4}{2}} - i\sqrt{\frac{\sqrt{41} + 4}{2}}$$
$$x_4 = -(\sqrt{\frac{\sqrt{41} - 4}{2}} - i\sqrt{\frac{\sqrt{41} + 4}{2}})$$

39 номер

Условие:

$z - i \leq 1$

Область:

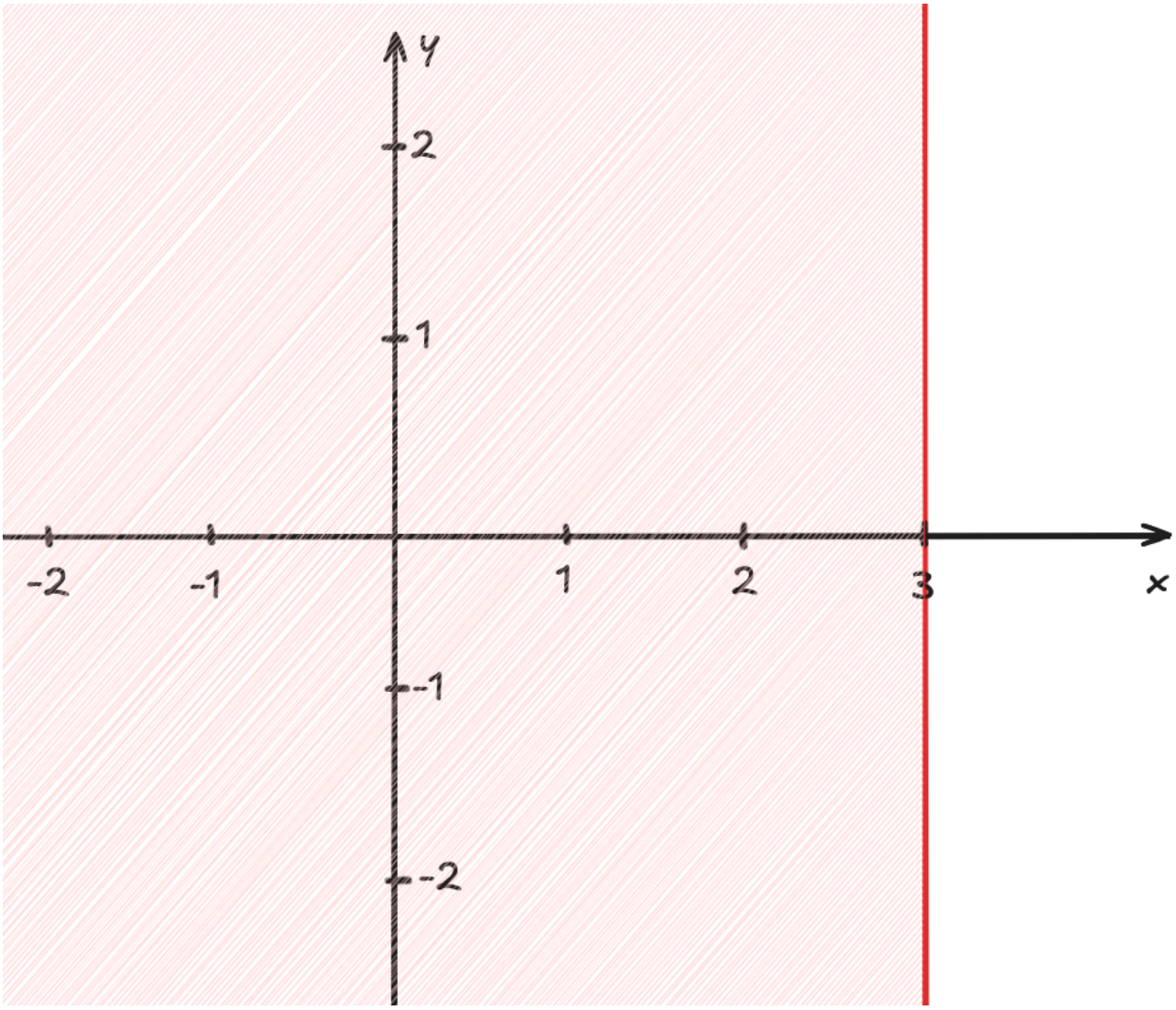


40 номер

Условие:

$\mathrm{Re}(z) \leq 3$

Область:

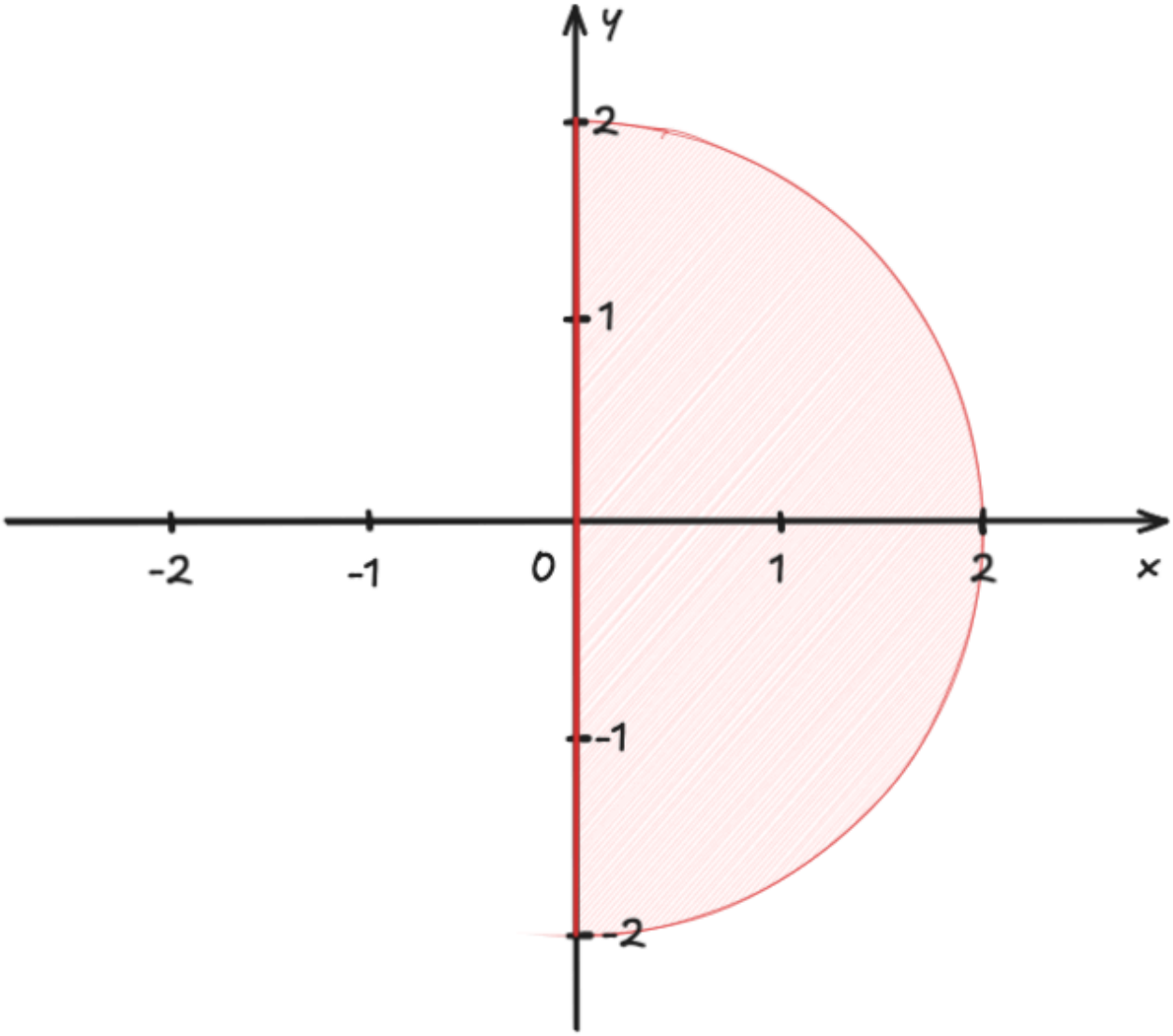


41 номер

Условие:

$z \leq 2 \text{ and } \operatorname{Re}(z) \geq 0$

Область:

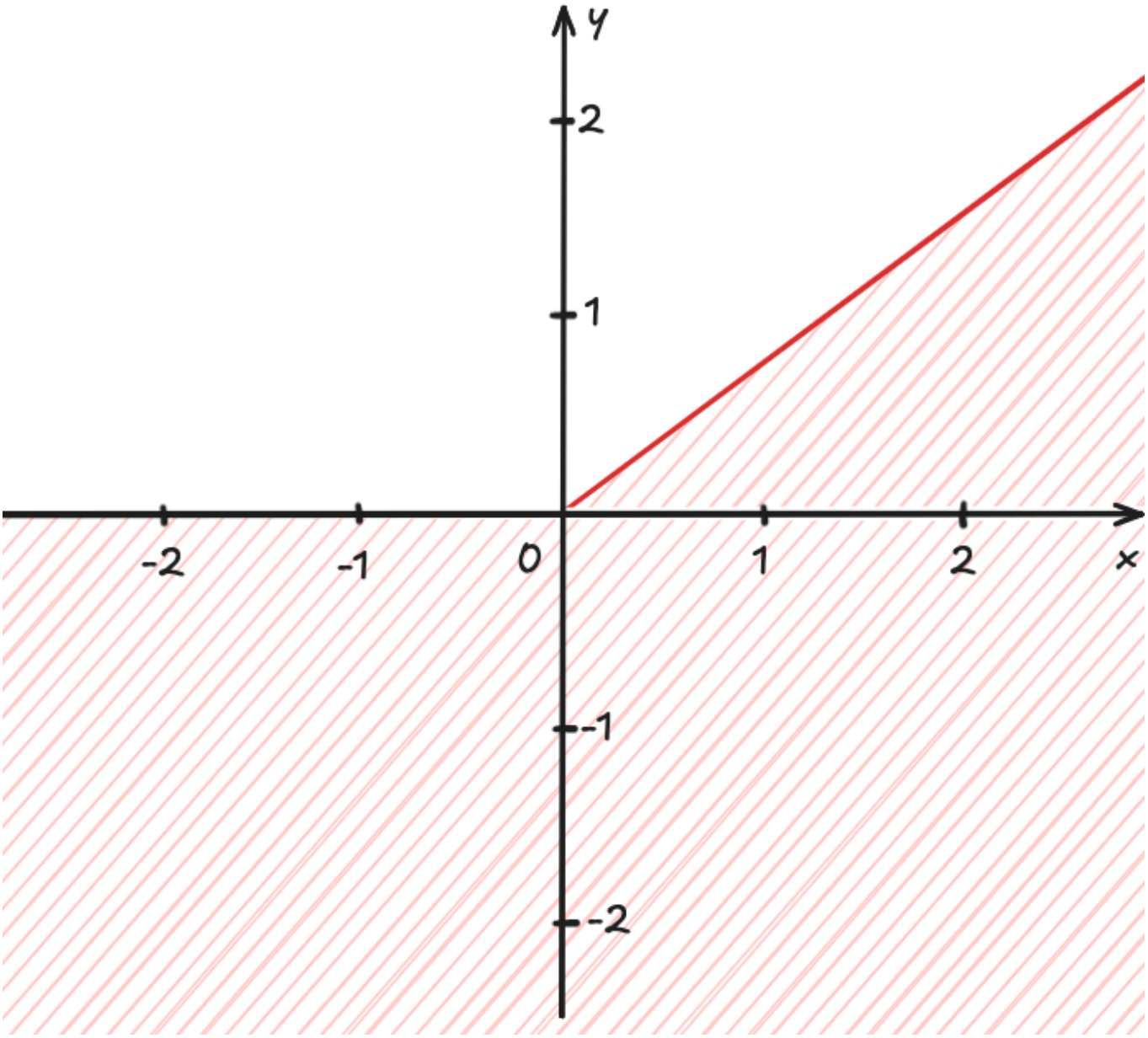


42 номер

Условие:

$arg(z) \leq \frac{\pi}{6}$

Область:

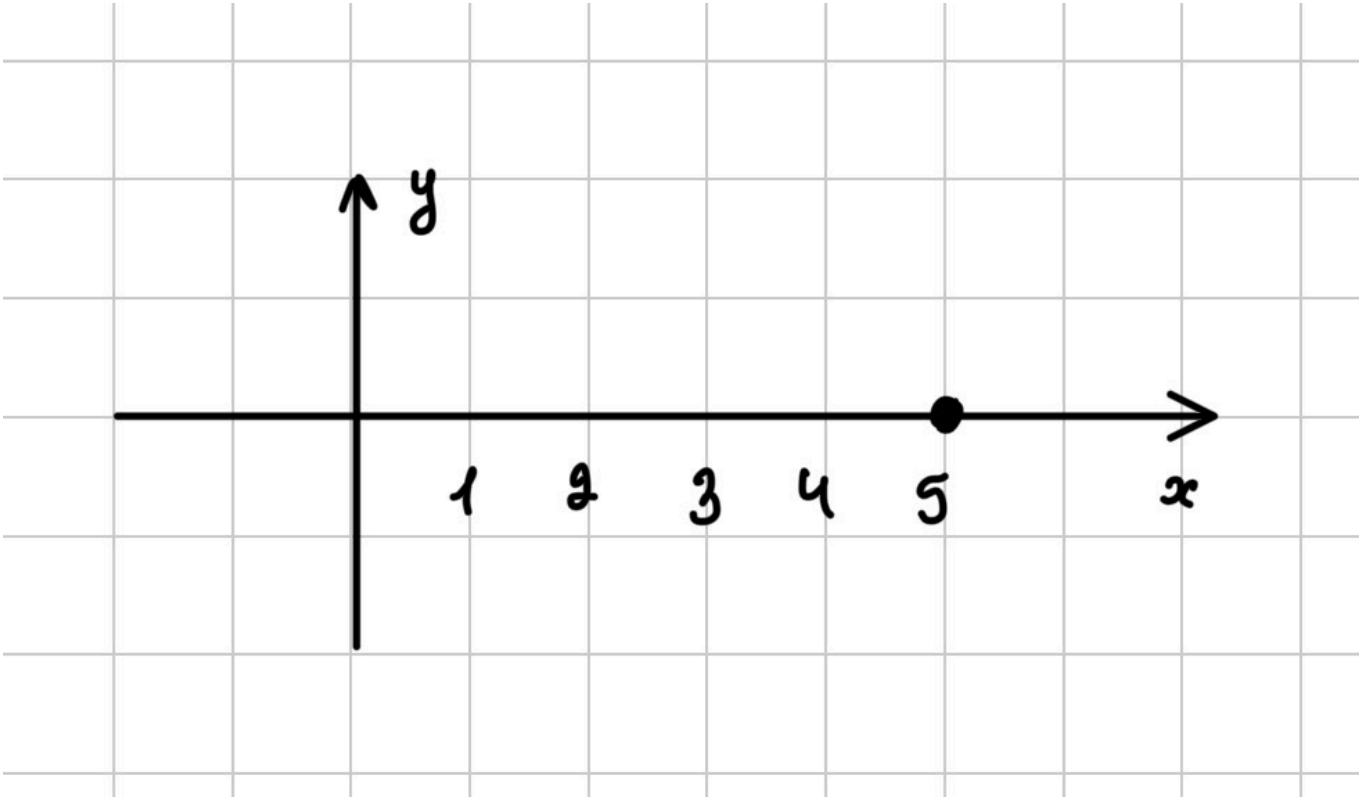


43 номер

Условие:

$z = 5; arg(z) \leq \frac{\pi}{3}$

Область:

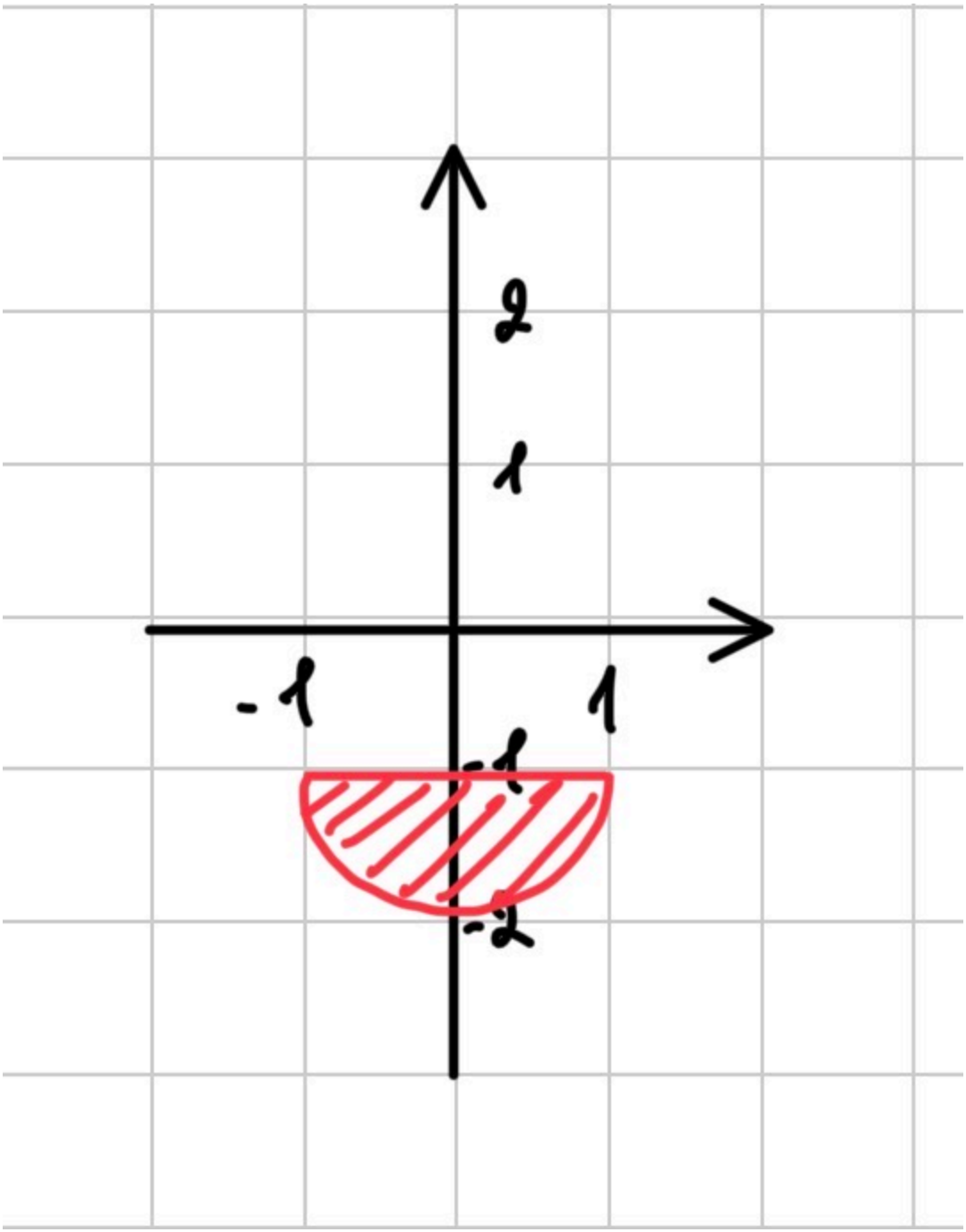


44 номер

Пример:

$z + i \leq 1; \mathrm{Im} \leq -1$

Решение:


2
1
-1
1
-1
-2